Chapter 3: Graphs of Quadratic Relations

Getting Started, page 130

1. a) Choices ii), v), and vi) all show linear relations, but the best choice is v), because it is the only choice that shows only a linear relation.

b) First differences are the differences between consecutive values of y. Only choice ii) shows these differences.

c) The distributive property occurs when each value in a set of brackets is multiplied by the value outside the brackets. This happens only in choice i).

d) A scatter plot shows many points on a graph. Only choice iv) shows this.

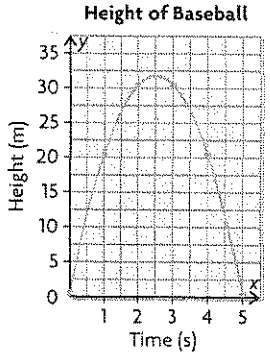
e) The intercepts are the points at which a graph crosses the x-axis and the y-axis. Only choice vi) shows these.

f) A line of symmetry is a line through a figure that divides the figure into two matching halves. Only choice iii) shows this.

2. a) Draw a line up from 15 on the age axis to meet the graph. Then go straight left from that point to determine the value of the y-coordinate. It is about 58, so the resting heartbeat of a person who is 15 years old will likely be about 58 beats per minute.

b) Draw a line right from 75 on the heart rate axis to meet the graph. Then go straight down from that point to determine the value of the x-coordinate. It is about 38, so a person with a resting heartbeat of about 75 beats will likely be about 38 years old.

3. a) Answers may vary, e.g.,



b) Draw a line up from 2.5 on the time axis to the graph. Then go left from that point to the height axis to determine the value of the height coordinate. It is about 32 m.

c) Draw a line right from 25 on the height axis to the graph. It meets the graph at two points. Go down from each point to determine the value of the time coordinates. They are about 1.4 s and 3.6 s.

4. a) $4(x + 3) = 4x + 12$

b) $2x(x - 5) = 2x^2 - 10x$

c) $-3x^2(x - 2) = -3x^3 + 6x^2$

d)
$4x(2x - 3) + 3x(7 - 5x)$
$= 8x^2 - 12x + 21x - 15x^2$
$= 8x^2 - 15x^2 - 12x + 21x$
$= -7x^2 + 9x$

e)
$7x^2(4x - 7 + 2x^2) - x(3x^2 - 5x - 2)$
$= 28x^3 - 49x^2 + 14x^4 - 3x^3 + 5x^2 + 2x$
$= 14x^4 + 28x^3 - 3x^3 + 5x^2 - 49x^2 + 2x$
$= 14x^4 + 25x^3 - 44x^2 + 2x$

f)
$-4x(x^3 - 3x^2) + 2x(5x^2 - 3x) - 6x^3(x - 3)$
$= -4x^4 + 12x^3 + 10x^3 - 6x^2 - 6x^4 + 18x^3$
$= -4x^4 - 6x^4 + 12x^3 + 10x^3 + 18x^3 - 6x^2$
$= -10x^4 + 40x^3 - 6x^2$

5. a) For $x = 1.5$:
$y = 2x - 3$
$y = 2(1.5) - 3$
$y = 0$

b) For $x = -3$:
$y = x^2$
$y = (-3)^2$
$y = 9$

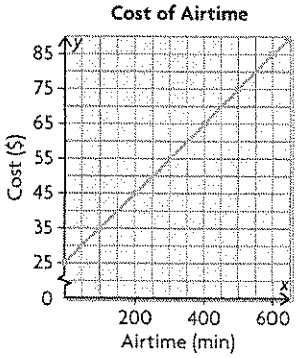
c) For $x = 4$:
$y = x^2 + 2x - 1$
$y = 4^2 + 2(4) - 1$
$y = 16 + 8 - 1$
$y = 23$

d) For $x = 2$:
$y = (2x + 1)(x - 3)$
$y = [2(2) + 1](2 - 3)$
$y = (4 + 1)(-1)$
$y = 5(-1)$
$y = -5$

6. For example, I calculated the cost for every one hundred minutes.

Airtime (min)	Cost ($)
0	25
100	35
200	45
300	55
400	65
500	75
600	85

I plotted these points on a grid and connected them with a straight line, because any number of minutes can be used, not just the numbers I picked.

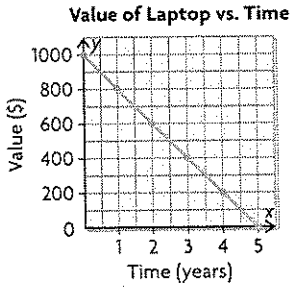


This graph is linear, so an equation for it will be in the form $y = mx + b$, where m is the slope of the line and b is the y-intercept. In this case, the cost for 0 minutes is $25, so the y-intercept is 25. The slope of the line is 0.10, so an equation is $C = 25 + 0.1t$, where t is the time in minutes and C is the cost in dollars.

7. For example, I calculated the value for every year.

Time (*years*)	Value ($)
0	1000
1	800
2	600
3	400
4	200
5	0

I plotted these points on a grid and connected them with a straight line, because the value of the laptop decreases gradually over the year.



This graph is linear, so an equation for it will be in the form $y = mx + b$, where m is the slope of the line and b is the y-intercept. In this case, the cost for 0 years is $1000, so the y-intercept is 1000. The slope of the line is –200, so an equation is $V = 1000 - 200t$, where t is the time in years and V is the value in dollars.

8. The x-intercept occurs where a graph crosses the x-axis. This graph crosses the x-axis at point (–4, 0), so this is the x-intercept. The y-intercept occurs where a graph crosses the y-axis. This graph crosses the y-axis at point (0, –8), so this is the y-intercept.

9. To calculate each x-intercept, set $y = 0$ and solve for x. To calculate each y-intercept, set $x = 0$ and solve for y.

a) x-intercept:

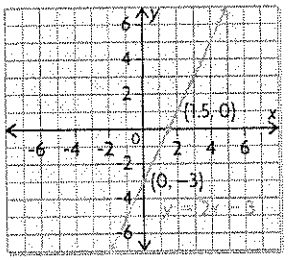
$y = 2x - 3$
$0 = 2x - 3$
$3 = 2x$
$x = 1.5$

The x-intercept is (1.5, 0).

y-intercept:

$y = 2x - 3$
$y = 2(0) - 3$
$y = -3$

The y-intercept is (0, –3).



b) x-intercept:

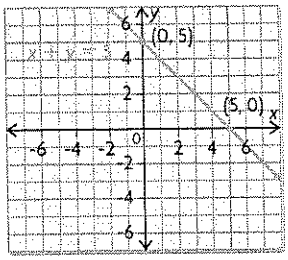
$x + y = 5$
$x + 0 = 5$
$x = 5$

The x-intercept is (5, 0).

y-intercept:

$x + y = 5$
$0 + y = 5$
$y = 5$

The y-intercept is (0, 5).



c) x-intercept:

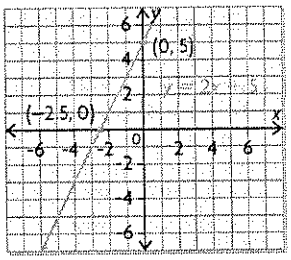
$y = 2x + 5$
$0 = 2x + 5$
$-5 = 2x$
$x = -2.5$

The x-intercept is (–2.5, 0).

y-intercept:

$y = 2x + 5$
$y = 2(0) + 5$
$y = 5$

The y-intercept is (0, 5).



d) x-intercept:

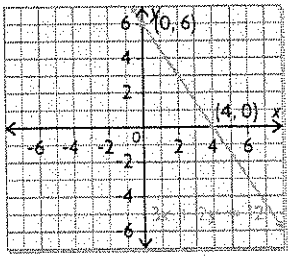
$3x + 2y = 12$
$3x + 2(0) = 12$
$3x = 12$
$x = 4$

The x-intercept is (4, 0).

y-intercept:

$3x + 2y = 12$
$3(0) + 2y = 12$
$2y = 12$
$y = 6$

The y-intercept is (0, 6).



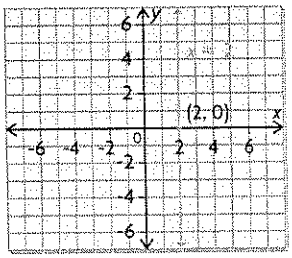
e) x-intercept:

$x = 2$

The x-intercept is (2, 0).

y-intercept:

It is not possible to set $x = 0$. There is no y-intercept.

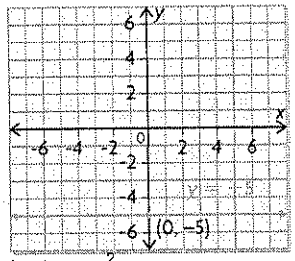


f) It is not possible to set $y = 0$. There is no x-intercept.

y-intercept:

$y = -5$

The y-intercept is (0, –5).



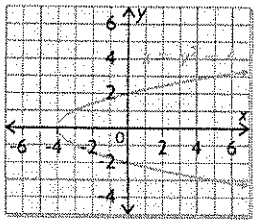
10. a) False; for example, look at $y = x^2$:

x	y	First Differences
1	1	3
2	4	5
3	9	7
4	16	

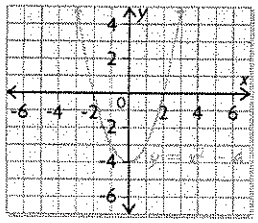
b) False; for example, look at $y = 2x$:

x	y	First Differences
1	2	
		2
2	4	
		2
3	6	
		2
4	8	

c) False; for example, look at the graph of $x = y^2 - 4$:



d) False; for example, look at the graph of $y = x^2 - 4$:



e) False; for example, look at this figure:

11. Answers may vary, e.g.,

Definition:
- Any relation that is not linear; any relation that has terms with power 2 or higher

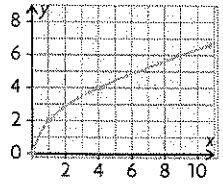
Characteristics:
- Its graph is not a straight line.
- First differences are not constant.

Nonlinear Relations

Examples:

$C = \pi r^2$

x	0	1	2	3	4
y	1	3	9	27	81



Non-examples:

$y = 3x - 4$

$x = 3$

$a = 2b + 3c - d$

Lesson 3.1: Exploring Quadratic Relations, page 136

1. a) No; this graph is linear, so it cannot represent a quadratic relation.
b) No; this graph has two "openings".
c) Yes; the graph opens upward in a U shape with a vertical line of symmetry.
d) Yes; the graph opens downward in a U shape with a vertical line of symmetry.
e) No; the two arms of the graph are straight, not curved.
f) No; the line segments are straight.
2. a) i) The degree of this relation is 1, because that is the value of the greatest exponent of x.
ii) The degree of this relation is 2, because that is the value of the greatest exponent of x.
iii) Expand:

$y = x(x + 4)$
$y = x^2 + 4$

The degree of this relation is 2 because that is the value of the greatest exponent of x.
iv) The degree of this relation is 3 because that is the value of the greatest exponent of x.
b) Relations ii) and iii) will have graphs that are parabolas, since they are degree 2.
3. Only relations ii) and iii) are quadratic. To determine the y-intercept, evaluate the relation for $x = 0$.
For relation ii):

$y = x^2 - 6x + 4$
$y = 0^2 - 6(0) + 4$
$y = 4$

The y-intercept is (0, 4).
For relation iii):

$y = x(x + 4)$
$y = 0(0 + 4)$
$y = 0$

The y-intercept is (0, 0).
4. a)

x	y	First Differences
10	21	
		41 – 21 = 20
20	41	
		61 – 41 = 20
30	61	
		81 – 61 = 20
40	81	

Since the first differences are the same, the relation is linear.

b)

x	y	First Differences	Second Differences
1	4	$7 - 4 = 3$	
2	7		$5 - 3 = 2$
3	12	$12 - 7 = 5$	
4	17		$5 - 5 = 0$
		$17 - 12 = 5$	

Since the first and second differences are different, the relation is nonlinear.

c)

x	y	First Differences	Second Differences
5	-2	$-3 + 2 = -1$	
6	-3		$-2 + 1 = -1$
7	-5	$-5 + 3 = -2$	
8	-8		$-3 + 2 = -1$
		$-8 + 5 = -3$	

Since the first differences are different, but the second differences are the same, then the relation is quadratic.

d)

x	y	First Differences	Second Differences
0	1	$-1 - 1 = -2$	
1	-1		$8 + 2 = 10$
2	7	$7 + 1 = 8$	
3	-11		$-18 - 8 = -26$
		$-11 - 7 = -18$	

Since the differences are not equal, the relation is nonlinear.

e)

x	y	First Differences	Second Differences
0	-2	$-1 + 2 = 1$	
1	-1		$7 - 1 = 6$
2	6	$6 + 1 = 7$	
3	25		$19 - 7 = 12$
		$25 - 6 = 19$	

Since the differences are not equal, the relation is nonlinear.

f)

x	y	First Differences	Second Differences
0	1	$2 - 1 = 1$	
1	2		$2 - 1 = 1$
2	4	$4 - 2 = 2$	
3	8		$4 - 2 = 4$
		$8 - 4 = 4$	
4	16		$8 - 4 = 4$
		$16 - 8 = 8$	

Since the differences are not equal, the relation is nonlinear.

5. a)

x	y	First Differences	Second Differences
-3	2.5	$5.0 - 2.5 = 2.5$	
-2	5.0		$1.5 - 2.5 = -1.0$
-1	6.5	$6.5 - 5.0 = 1.5$	
0	7.0		$0.5 - 1.5 = -1.0$
		$7.0 - 6.5 = 0.5$	

The second differences are negative, so the parabola opens downward.

b)

x	y	First Differences	Second Differences
-2	0	$-5 - 0 = -5$	
-1	-5		$5 - (-5) = 10$
0	0	$0 - (-5) = 5$	
1	15		$15 - 5 = 10$
2	40	$15 - 0 = 15$	$25 - 15 = 10$
		$40 - 15 = 25$	

The second differences are positive, so the parabola opens upward.

c)

x	y	First Differences	Second Differences
-2	-3	$3 - (-3) = 6$	
-1	3		$2 - 6 = -4$
0	5	$5 - 3 = 2$	
1	3		$-2 - 2 = -4$
2	-3	$3 - 5 = -2$	$-6 - (-2) = -4$
		$-3 - 3 = -6$	

The second differences are negative, so the parabola opens downward.

d)

x	y	First Differences	Second Differences
0	–1	4 – (–1) = 5	
1	4		11 – 5 = 6
		15 – 4 = 11	
2	15		17 – 11 = 6
		32 – 15 = 17	
3	32		23 – 17 = 6
		55 – 32 = 23	
4	55		

The second differences are positive, so the parabola opens upward.

6. a) Upward, since the coefficient of x^2 is positive.
b) Downward, since the coefficient of x^2 is negative.
c) Downward, since the coefficient of x^2 is negative.
d) Upward, since the coefficient of x^2 is positive.

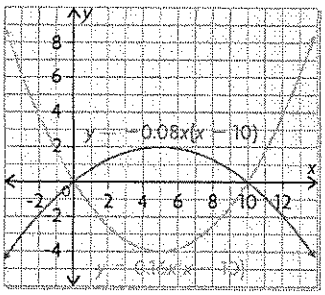
7. The condition $a \neq 0$ is needed because otherwise there would be no x^2 term and the relation would be linear.

Lesson 3.2: Properties of Graphs of Quadratic Relations, page 145

1. a) *y*-intercept: 0; zeros: –4, 0; vertex: (–2, 4); axis of symmetry: $x = -2$
b) *y*-intercept: –3; zeros: –2, 2; vertex: (0, –3); axis of symmetry: $x = 0$
c) *y*-intercept: 0; zeros: 0, 4; vertex: (2, 4); axis of symmetry: $x = 2$

2. The maximum or minimum is always the highest or lowest *y*-value on the graph.
a) maximum value: 4
b) minimum value: –3
c) maximum value: 4

3. For both graphs the axis of symmetry is halfway between the zeros, at $x = (0 + 10) \div 2 = 5$. One graph has a maximum of (5, 2) and the other a minimum of (5, –4).



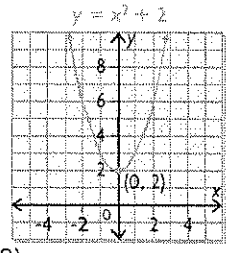
4. All values can be read directly from the graph.
a) i) (3, 2) **ii)** 0, 6 **iii)** $x = 3$
iv) The second differences would be negative, because the curve opens downward.
b) i) (2, –3) **ii)** 0, 4 **iii)** $x = 2$
iv) The second differences would be positive, because the curve opens upward.

5. a) The vertex is a minimum because the curve opens upward (the second differences are positive).
b) Since the vertex is a minimum, it must occur below the zeros. Then, the *y*-value must be negative
c) The *x*-value must be halfway between the zeros, $x = (0 + 6) \div 2 = 3$.

6. All values can be read directly from the graph.
a) i) $x = -4$
ii) The vertex is (–4, –8).
iii) When $x = 0$, $y = 0$, therefore the *y*-intercept is 0.
iv) When $y = 0$, $x = -8$ and 0.
The *x*-intercepts are –8 and 0.
v) –8 is the minimum value
b) i) $x = 2$
ii) The vertex is (2, 2).
iii) When $x = 0$, $y = 0$, therefore the *y*-intercept is 0.
iv) When $y = 0$, $x = 0$ and 4.
The *x*-intercepts are 0 and 4.
v) 2 is the maximum value
c) i) $x = 0$
ii) The vertex is (0, 8).
iii) When $x = 0$, $y = 8$. The *y*-intercept is 8.
iv) When $y = 0$, $x = -6$ and 6.
The *x*-intercepts are –6 and 6.
v) 8 is the maximum value.

7. a)

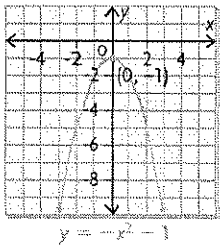
x	y
–2	6
–1	3
0	2
1	3
2	6



i) $x = 0$ **ii)** The vertex is (0, 2).
iii) From the vertex, when $x = 0$, $y = 2$.
The *y*-intercept is 2.
iv) Since the graph is opening upward and its vertex is above the *x*-axis there are no zeros.
v) 2 is the minimum value

b)

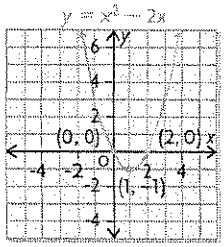
x	y
–2	–5
–1	–2
0	–1
1	–2
2	–5



i) $x = 0$ **ii)** The vertex is (0, –1).
iii) From the vertex, when $x = 0$, $y = -1$.
The *y*-intercept is –1.
iv) Since the graph is opening downward and its vertex is below the *x*-axis there are no zeros.
v) –1 is the maximum value

c)

x	y
–2	8
–1	3
0	0
1	–1
2	0

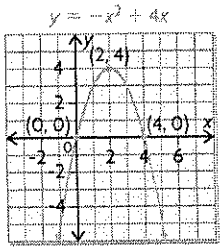


i) $x = 1$ **ii)** The vertex is (1, –1).

iii) From the graph, when $x = 0$, $y = 0$.
The y-intercept is 0.
iv) When $y = 0$, $x = 0$ and 2.
The x-intercepts are 0 and 2.
v) –1 is the minimum value

d)

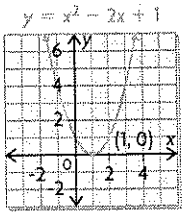
x	y
–1	–5
0	0
1	3
2	4
3	3
4	0



i) $x = 2$ **ii)** The vertex is (2, 4).
iii) From the graph, when $x = 0$, $y = 0$.
The y-intercept is 0.
iv) When $y = 0$, $x = 0$ and 4.
The x-intercepts are 0 and 4.
v) 4 is the maximum value

e)

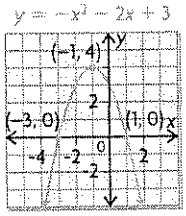
x	y
–1	4
0	1
1	0
2	1
3	4



i) $x = 1$ **ii)** The vertex is (1, 0).
iii) From the graph, when $x = 0$, $y = 1$.
The y-intercept is 1.
iv) From the vertex, when $y = 0$, $x = 1$.
The x-intercept is 1.
v) 0 is the minimum value

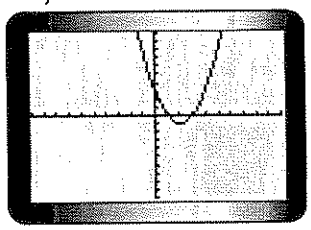
f)

x	y
–3	0
–2	3
–1	4
0	3
1	0



i) $x = -1$ **ii)** The vertex is (–1, 4).
iii) From the graph, when $x = 0$, $y = 3$.
The y-intercept is 3.
iv) From the graph, when $y = 0$, $x = -3$ and 1.
The x-intercepts are –3 and 1.
v) 4 is the maximum value

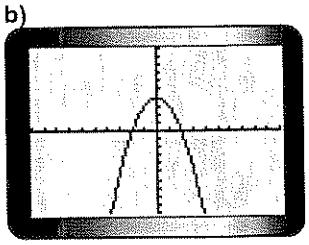
8. a)



i) From the graph, $x = 3$ and $x = 1$ are equal distance from the axis of symmetry.
$x = (3 + 1) \div 2$
$x = 2$

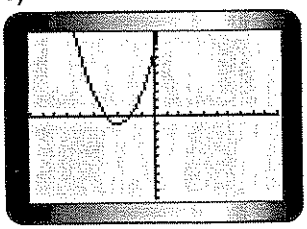
ii) The vertex is the point where the axis of symmetry passes through the parabola.
$y = x^2 - 4x + 3$
$y = 2^2 - 4(2) + 3$
$y = 4 - 8 + 3$
$y = -1$
The vertex is (2, –1).
iii) In standard form, the y-intercept is the c-value.
The y-intercept is 3.
iv) From the graph, the zeros are 1 and 3.
v) From the vertex, the minimum value is –1.

b)



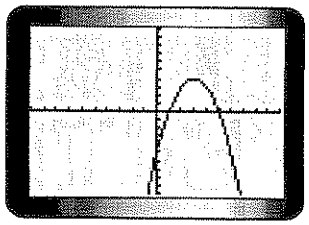
i) From the graph, $x = 2$ and $x = -2$ are equal distance from the axis of symmetry.
$x = (-2 + 2) \div 2$
$x = 0$
ii) The vertex is the point where the axis of symmetry passes through the parabola.
$y = -x^2 + 4$
$y = -0^2 + 4$
$y = 4$
The vertex is (0, 4).
iii) From the vertex, when $x = 0$, then $y = 4$. Therefore the y-intercept is 4.
iv) From the graph, the zeros are –2 and 2.
v) From the vertex, the maximum value is 4.

c)

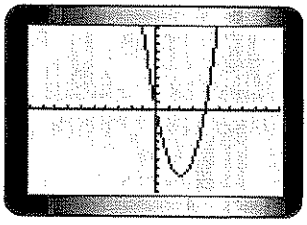


i) From the graph, $x = -4$ and $x = -2$ are equal distance from the axis of symmetry.
$x = (-2 - 4) \div 2$
$x = -3$
ii) The vertex is the point where the axis of symmetry passes through the parabola.
$y = x^2 + 6x + 8$
$y = (-3)^2 + 6(-3) + 8$
$y = 9 - 18 + 8$
$y = -1$
The vertex is (–3, –1).
iii) The y-intercept is the c-value in standard form. The y-intercept is 8.
iv) From the graph, the zeros are –4 and –2.
v) From the vertex, the minimum value is – 1.

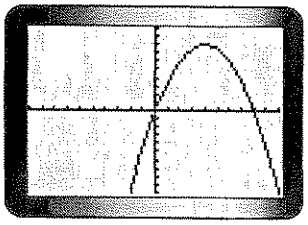
d)



i) From the graph, $x = 1$ and $x = 5$ are equal distance from the axis of symmetry.
$x = (1 + 5) \div 2$
$x = 3$
ii) The vertex is the point where the axis of symmetry passes through the parabola.
$y = -x^2 + 6x - 5$
$y = -(3)^2 + 6(3) - 5$
$y = -9 + 18 - 5$
$y = 4$
The vertex is (3, 4).
iii) The y-intercept is the c-value in standard form. The y-intercept is –5.
iv) From the graph, the zeros are 1 and 5.
v) From the vertex, the maximum value is 4.
e)



i) From the factored equation, the zeros, $x = 0$ and $x = 4$, are equal distance from the axis of symmetry.
$x = (0 + 4) \div 2$
$x = 2$
ii) The vertex is the point where the axis of symmetry passes through the parabola.
$y = 2x(x - 4)$
$y = 2(2)(2 - 4)$
$y = -8$
The vertex is (2, –8).
iii) When $x = 0$ then
$y = 2x(x - 4)$
$y = 2(0)(0 - 4)$
$y = 0$
The y-intercept is 0.
iv) From the factored form, the zeros are 0 and 4.
v) From the vertex, the minimum value is –8.
f)



i) From the graph, $x = 0$ and $x = 8$ are equal distance from the axis of symmetry.
$x = (0 + 8) \div 2$
$x = 4$
ii) The vertex is the point where the axis of symmetry passes through the parabola.
$y = -0.5x(x - 8)$
$y = -0.5(4)(4 - 8)$
$y = -2(-4)$
$y = 8$
The vertex is (4, 8).
iii) When $x = 0$ then
$y = -0.5x(x - 8)$
$y = -0.5(0)(0 - 8)$
$y = 0$
The y-intercept is 0.
iv) From the graph, the x-intercepts are 0 and 8.
v) From the vertex, the maximum value is 8.

9. a) $x = \dfrac{3+9}{2}$

$x = \dfrac{12}{2}$

$x = 6$

The axis of symmetry is $x = 6$.

b) $x = \dfrac{-18+7}{2}$

$x = \dfrac{-11}{2}$

$x = -5.5$

The axis of symmetry is $x = -5.5$.

c) $x = \dfrac{-5.25+3.75}{2}$

$x = \dfrac{-1.5}{2}$

$x = -0.75$

The axis of symmetry is $x = -0.75$.

d) $x = \dfrac{-4.5-1.5}{2}$

$x = \dfrac{-6}{2}$

$x = -3$

The axis of symmetry is $x = -3$.

10. The two points are an equal distance away from the axis of symmetry, so they can be used to determine the equation.

$x = \dfrac{-1+5}{2}$

$x = \dfrac{4}{2}$

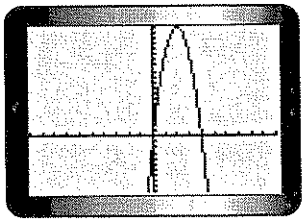
$x = 2$

To find the vertex, solve for y when $x = 2$.
$y = 4x^2 - 16x + 21$
$y = 4(2)^2 - 16(2) + 21$
$y = 16 - 32 + 21$
$y = 5$

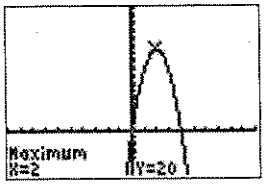
11. a) Disagree, not all parabolas intersect the x-axis at two points. For example, $y = 2x^2 - 4x + 2$ is a parabola of that form, but it does not cross the x-axis.
b) Agree, all parabolas intersect the y-axis once. Since parabolas of this form have a domain of all x-values then there must be a point where $x = 0$, thus the parabola intercepts the y-axis.
c) Disagree, all parabolas that open downward have negative (constant) second differences.
12. a) To determine the zeros, let $h = 0$.

$h = 20t - 5t^2$
$0 = 20t - 5t^2$
$0 = t(20 - 5t)$

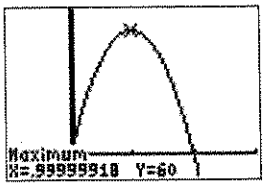
$t = 0$ or $5t = 20$
The zeros are 0 and 4. The football starts on the ground at 0 s and will hit the ground at 4 s.
b) The t-value of the vertex is midway between the zeros, $t = \frac{(0+4)}{2} = 2$.
When $t = 2$ then:
$h = 20t - 5t^2$
$h = 20(2) - 5(2)^2$
$h = 40 - 20 = 20$
The vertex is (2, 20).
c)



d) From my calculations above, the vertex is (2, 20), so the maximum height should be 20 m at 2 s. I used the calculator to check.

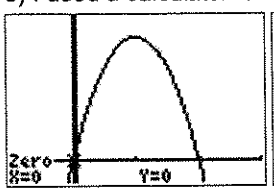


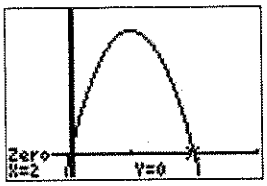
13. a) The maximum profit is the P value of the vertex. I used the calculator's maximum function to determine the vertex, which is (1, 60), so the maximum profit is $60 000.



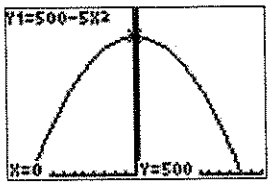
b) The number of MP3 players sold in the x-value of the vertex. There are 1000 MP3 players sold when the profit is at its maximum.

c) I used a calculator to determine the zeros.

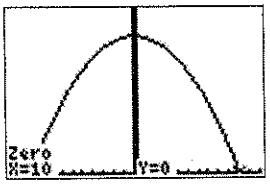




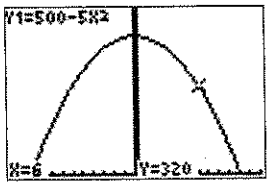
$P = 0$ when $x = 0$ and $x = 2$. Therefore there are two break-even points, 0 and 2000.
14. a) The height of the helicopter is the same as the height of the raft before it is dropped. I used the calculator's value function to determine the height when $x = 0$.



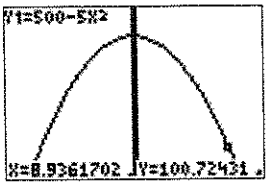
The helicopter is 500 m above the water.
b) The life raft reaches the water when y = 0. I used the calculator's zero function to determine the zero.



The life raft reaches the water 10 s after it is dropped.
c) Use the calculator's value function to determine y when $x = 6$:

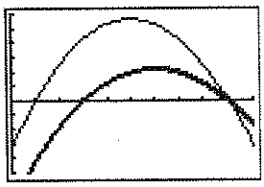


The raft is 320 m above the water.
d) Use the calculator's value function to determine x when $y = 100$:



The raft is 100 m above the water at about 8.9 s.

15. Solve using graphing technology. In the graph, this year's curve is a thin line, last year's is a thick line. The profits increased (or losses decreased) if fewer than 900 000 game players were sold since the y-value of the line representing this year's profit is higher for $x < 9$. If more than 900 000 game players were sold, then losses increased.



16. a) If a is negative the quadratic relation has a maximum, but if a is positive the parabola has a minimum.
b) The y-value of the vertex is the maximum or minimum value of the parabola, and the x-value is the point at which it occurs.
17. a) i) $x = 2y^2$

x	y	First Differences	Second Differences
1	2	8 – 2 = 6	
2	8		10 – 6 = 4
3	18	18 – 8 = 10	14 – 10 = 4
4	32	32 – 18 = 14	18 – 14 = 4
5	50	50 – 32 = 18	

ii) $y = 2^x$

x	y	First Differences	Second Differences
1	2	4 – 2 = 2	
2	4		4 – 2 = 2
3	8	8 – 4 = 4	8 – 4 = 4
4	16	16 – 8 = 8	

iii) $y = x^3$

x	y	First Differences	Second Differences
1	1	8 – 1 = 7	
2	8		19 – 6 = 12
3	27	27 – 8 = 19	37 – 19 = 18
4	64	64 – 27 = 37	

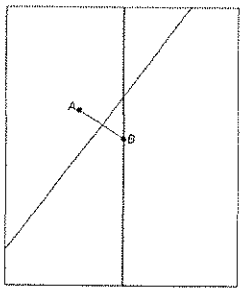
iv) $y = 2x^4$

x	y	First Differences	Second Differences
1	2	32 – 2 = 30	
2	32		130 – 30 = 100
3	162	162 – 32 = 130	350 – 130 = 220
4	512	512 – 162 = 350	

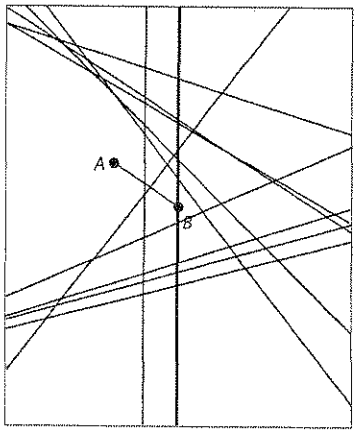
b) Yes, the relation in i) represents a parabola because it has constant second differences.
c) Yes, the relation in i) is quadratic because it has constant second differences.
18. Answers may vary, e.g., I looked at several examples and compared the x-coordinate of the vertex with the coefficient of x:
$y = 5x^2 - 4.2x + 8$; 0.42, negative one tenth of the coefficient of x
$y = 2x^2 - 3.2x + 8$; 0.64, not negative one tenth of the coefficient of x
$y = 5x^2 - 3.2x + 6$; 0.32, negative one tenth of the coefficient of x
I made the hypothesis that the x-coordinate of the vertex is equal to negative one tenth of the x coefficient whenever the coefficient of x^2 equals 5. To prove it, I investigated the equation $y = 5x^2 + bx$, where $c = 0$.

Since the zeros of the equation are 0 and $-\frac{b}{5}$, the x-coordinate of the vertex is $-\frac{b}{10}$. Different values of c will change the y-intercept, but not the line of symmetry or the x-coordinate of the vertex.

Curious Math, page 149
1.– 6.



The crease in the paper intersects AB at its midpoint.
7.– 9.



The parabola is formed around point A, which lies on the axis of symmetry. The parabola is located entirely on one side of the line passing through point B.

Lesson 3.3: Factored Form of a Quadratic Relation, page 155

1. a) i) The zeros occur when $-2x = 0$ or $x + 3 = 0$. Therefore, the zeros are $x = 0$ and $x = -3$.

ii) The zeros are the values of x that make each factor equal 0. If one factor equals zero then $y = 0$, which means that value of x is a zero.

iii) The y-intercept is when $x = 0$.

$$y = -2x(x + 3)$$
$$y = -2(0)(0 + 3)$$
$$y = 0$$

iv) The axis of symmetry is midway between the zeros.

$$x = \frac{0 - 3}{2}$$
$$= -1.5$$

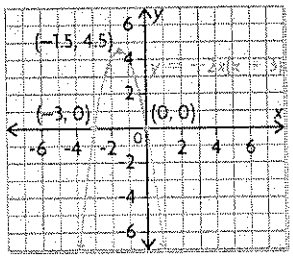
v) The vertex is the point where the axis of symmetry intersects the parabola: $x = -1.5$.

$$y = -2x(x + 3)$$
$$y = -2(-1.5)(-1.5 + 3)$$
$$y = 4.5$$

The vertex is $(-1.5, 4.5)$.

vi) Yes, since if I were to expand the equation, the highest power of x would be 2.

vii)



b) i) $x - 3 = 0$ and $x + 1 = 0$. Therefore, the zeros are $x = -1$ and $x = 3$.

ii) The zeros are the values of x where a factor equals 0.

iii) The y-intercept is when $x = 0$.

$$y = (x - 3)(x + 1)$$
$$y = (0 - 3)(0 + 1)$$
$$y = -3$$

iv) The axis of symmetry is midway between the zeros.

$$x = \frac{3 - 1}{2}$$
$$= 1$$

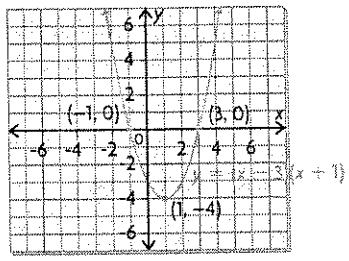
v) The vertex is the point at which $x = 1$.

$$y = (x - 3)(x + 1)$$
$$y = (1 - 3)(1 + 1)$$
$$y = -4$$

The vertex is $(1, -4)$.

vi) Yes, since if I were to expand the equation the highest power of x would be 2.

vii)



c) i) $x - 1 = 0$ and $x + 2 = 0$. Therefore, the zeros are $x = 1$ and $x = -2$.

ii) The zeros are the values of x where a factor equals 0.

iii) The y-intercept is when $x = 0$.

$$y = 2(x - 1)(x + 2)$$
$$y = 2(0 - 1)(0 + 2)$$
$$y = -4$$

iv) The axis of symmetry is midway between the zeros.

$$x = \frac{1 - 2}{2}$$
$$= -0.5$$

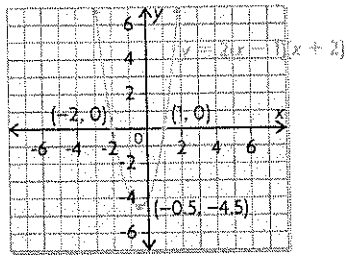
v) The vertex the point at which the axis of symmetry intersects the parabola, where $x = -0.5$.

$$y = 2(x - 1)(x + 2)$$
$$y = 2(-0.5 - 1)(-0.5 + 2)$$
$$y = -4.5$$

The vertex is $(-0.5, -4.5)$.

vi) Yes, since if I were to expand the equation the highest power I would get is x^2.

vii)



2. a) The zeros of the equation are 2 and –3. Therefore, the graph is either ii) or v). Since these graphs have different y-intercepts, I can determine the proper graph by determining the y-intercept.

$$y = (x - 2)(x + 3)$$
$$y = (0 - 2)(0 + 3)$$
$$y = -6$$ The correct graph is ii).

b) The zeros of the equation are –2 and 3. Therefore, the graph is either iv) or vi). Since these graphs have different y-intercepts I can determine the proper graph by determining the y-intercept.

$$y = (x - 3)(x + 2)$$
$$y = (0 - 3)(0 + 2)$$
$$y = -6$$ The correct graph is iv).

c) The zeros of the equation are –2 and –3. Therefore, graph i) is the correct parabola.

d) The zeros of the equation are –2 and 3. Therefore, graph vi) is the correct parabola since iv) matches equation b).

e) The zeros of the equation are 2 and –3. Therefore, graph v) is the correct parabola since ii) matches equation a).
f) The zeros of the equation are 2 and 3. Therefore, graph iii) is the correct parabola.

3. Enter the zeros using the factored form of the equation, $y = a(x - 2)(x + 6)$. Now, solve the equation for a by substituting in the point (3, 5).

$y = a(x-2)(x+6)$
$5 = a(3-2)(3+6)$
$5 = 9a$

$$a = \frac{5}{9}$$

4. Determine the y-intercept by setting $x = 0$ and solving for y. Determine the zeros by setting each factor equal to 0 and solving for x. The axis of symmetry is midway between the two zeros. The x-value of the vertex is the axis of symmetry, and the y-value can be solved for by evaluating the equation at the axis of symmetry.

a) y-intercept:
$y = (x - 3)(x + 3)$
$y = (0 - 3)(0 + 3)$
$y = -9$

zeros:
$x - 3 = 0$ and $x + 3 = 0$
$x = -3, 3$

axis of symmetry:
$$x = \frac{3-3}{2}$$
$x = 0$

vertex:
$y = (x - 3)(x + 3)$
$y = (0 - 3)(0 + 3)$
$y = -9$

The vertex is (0, –9).

b) y-intercept:
$y = (x + 2)(x + 2)$
$y = (0 + 2)(0 + 2)$
$y = 4$

zeros:
$x + 2 = 0$ and $x + 2 = 0$
$x = -2$

axis of symmetry:
$$x = \frac{-2-2}{2}$$
$x = -2$

vertex:
$y = (x + 2)(x + 2)$
$y = (-2 + 2)(-2 + 2)$
$y = 0$

The vertex is (–2, 0).

c) y-intercept:
$y = (x - 2)(x - 2)$
$y = (0 - 2)(0 - 2)$
$y = 4$

zeros:
$x - 2 = 0$ and $x - 2 = 0$
$x = 2$

axis of symmetry:
$$x = \frac{2+2}{2}$$
$x = 2$

vertex:
$y = (x - 2)(x - 2)$
$y = (2 - 2)(2 - 2)$
$y = 0$

The vertex is (2, 0).

d) y-intercept:
$y = -(x - 2)(x + 2)$
$y = -(0 - 2)(0 + 2)$
$y = 4$

zeros:
$x - 2 = 0$ and $x + 2 = 0$
$x = -2, 2$

axis of symmetry:
$$x = \frac{2-2}{2}$$
$x = 0$

vertex:
$y = -(x - 2)(x + 2)$
$y = -(0 - 2)(0 + 2)$
$y = 4$

The vertex is (0, 4).

e) y-intercept:
$y = 2(x + 3)(x + 3)$
$y = 2(0 + 3)(0 + 3)$
$y = 18$

zeros:
$x + 3 = 0$ and $x + 3 = 0$
$x = -3$

axis of symmetry:
$$x = \frac{-3-3}{2}$$
$x = -3$

vertex:
$y = 2(x + 3)(x + 3)$
$y = 2(-3 + 3)(-3 + 3)$
$y = 0$

The vertex is (–3, 0).

f) y-intercept:
$y = -4(x - 4)(x - 4)$
$y = -4(0 - 4)(0 - 4)$
$y = -64$

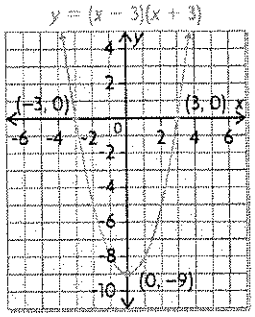
zeros:
$x - 4 = 0$ and $x - 4 = 0$
$x = 4$

axis of symmetry:
$$x = \frac{4+4}{2}$$
$x = 4$

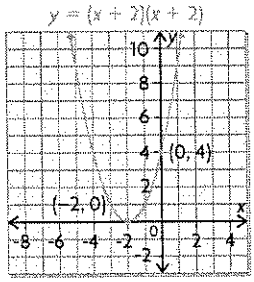
vertex:
$y = -4(x - 4)(x - 4)$
$y = -4(4 - 4)(4 - 4)$
$y = 0$

The vertex is (4, 0).

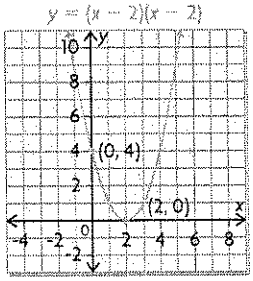
5. a)



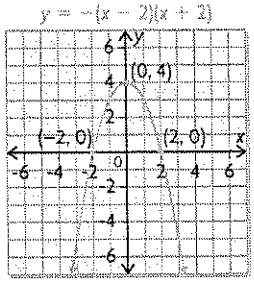
b)



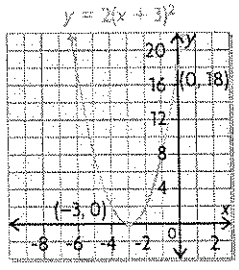
c)



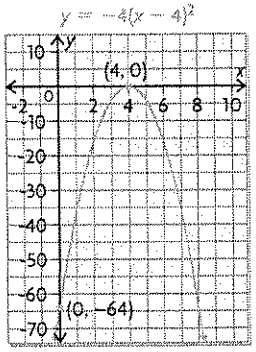
d)



e)



f)



6. To determine the value of a, insert the zeros and the y-intercept into the general factored form and solve for a.

a) $y = a(x-r)(x-s)$
$y = a(x-4)(x-2)$
$1 = a(0-4)(0-2)$
$1 = 8a$
$\frac{1}{8} = a$

b) $y = a(x-r)(x-s)$
$y = a(x-4)(x+2)$
$-1 = a(0-4)(0+2)$
$-1 = -8a$
$\frac{1}{8} = a$

c) The minimum value occurs at the vertex.
The y-value is -10 and the x-value is $x = (0 + 5) \div 2 = 2.5$.
$y = a(x-r)(x-s)$
$y = a(x-5)(x-0)$
$-10 = a(2.5-5)(2.5-0)$
$-10 = -6.25a$
$\frac{-10}{-6.25} = \frac{-6.25a}{-6.25}$
$1.6 = a$

d) The maximum value occurs at the vertex.
The y-value is 6 and the x-value is $x = (-3 + 5) \div 2 = 1$.
$y = a(x-r)(x-s)$
$y = a(x-5)(x+3)$
$6 = a(1-5)(1+3)$
$6 = -8a$
$\frac{6}{-8} = \frac{-8a}{-8}$
$-\frac{3}{8} = a$

e) The vertex is on the x-axis, so it is also the zero.
$y = a(x-r)(x-s)$
$y = a(x-5)(x-5)$
$-10 = a(0-5)(0-5)$
$-10 = -25a$
$\frac{-10}{-25} = \frac{-25a}{-25}$
$-0.4 = a$

7. a) From the graph, the zeros are $x = -40$ and 40, the y-intercept is 40, and the vertex is (0, 40). The axis of symmetry is $x = 0$. To determine the equation, insert the zeros and the y-intercept into the general factored form and solve for a.

$$y = a(x - r)(x - s)$$
$$y = a(x - 40)(x + 40)$$
$$40 = a(0 - 40)(0 + 40)$$
$$a = -\frac{1}{40}$$

The equation is $y = -\frac{1}{40}(x - 40)(x + 40)$.

b) From the graph, the zeros are $x = 10$ and 30, the y-intercept is 30, and the vertex is (20, –10). The axis of symmetry is $x = 20$. To determine the equation, insert the zeros and the y-intercept into the general factored form and solve for a.

$$y = a(x - r)(x - s)$$
$$y = a(x - 10)(x - 30)$$
$$-10 = a(20 - 10)(20 - 30)$$
$$a = \frac{1}{10}$$

The equation is $y = \frac{1}{10}(x - 10)(x - 30)$.

c) From the graph, the zeros are $x = -4$ and 1, and the vertex is (–1.5, –2). The axis of symmetry is $x = -1.5$. To determine the equation, insert the zeros and the vertex into the general factored form and solve for a.

$$y = a(x - r)(x - s)$$
$$y = a(x - 1)(x + 4)$$
$$-2 = a(-1.5 - 1)(-1.5 + 4)$$
$$-2 = -6.25a$$
$$\frac{-2}{-6.25} = \frac{-6.25a}{-6.25}$$
$$0.32 = a$$

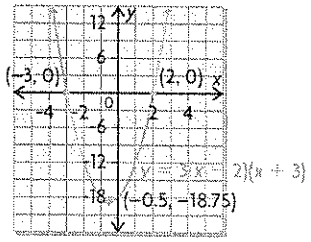
The equation is $y = 0.32(x + 4)(x - 1)$.

d) From the graph, the zeros are $x = -1$ and –5, and the vertex is (–3, 3.5). The axis of symmetry is $x = -3$. To determine the equation, insert the zeros and the vertex into the general factored form and solve for a.

$$y = a(x - r)(x - s)$$
$$y = a(x + 1)(x + 5)$$
$$3.5 = a(-3 + 1)(-3 + 5)$$
$$3.5 = -4a$$
$$\frac{3.5}{-4} = \frac{-4a}{-4}$$
$$-0.875 = a$$

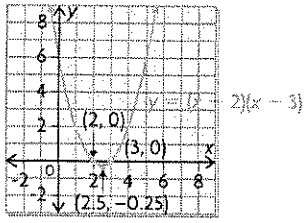
The equation is $y = -0.875(x + 1)(x + 5)$.

8. a)



b) When $a = 2$ the vertex shifts up and the curve becomes wider. When $a = 1$, the vertex shifts up farther and the curve widens more. When $a = 0$, we get a straight line along the x-axis. When a is negative, the curve is reflected in the x-axis, but the shape of the parabola is the same as when a is positive.

9. a)



b) The second zero would move from 3 to match each value of s. The farther the second zero was from 2, the lower the vertex would shift. When $s = 2$ then the vertex would sit on the x-axis.

10. a) Since the parabola crosses the y-axis at –75, then (0, –75) is a point on the parabola. To determine the equation, insert the zeros and the y-intercept into the general factored form and solve for a.

$$y = a(x - r)(x - s)$$
$$y = a(x + 3)(x - 5)$$
$$-75 = a(0 + 3)(0 - 5)$$
$$-75 = -15a$$
$$a = 5$$

The equation is $y = 5(x + 3)(x - 5)$.

b) The axis of symmetry is

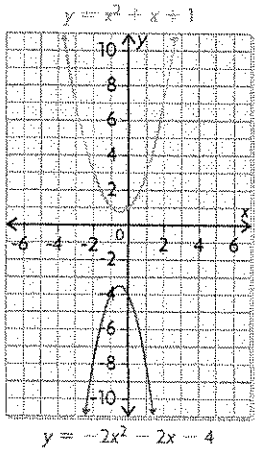
$$x = \frac{-3 + 5}{2}$$
$$x = 1$$

When $x = 1$ then

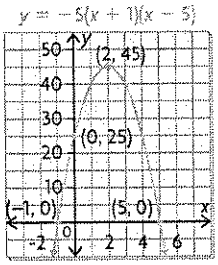
$$y = 5(x + 3)(x - 5)$$
$$y = 5(1 + 3)(1 - 5)$$
$$y = 5(4)(-4)$$
$$y = -80$$

The vertex is (1, –80).

11. The equation $y = a(x - r)(x - s)$ cannot be used if there are no zeros. This occurs when the quadratic form cannot be factored using real numbers. Here are two examples.



12. a), b) Since the ball starts its flight 25 m high, that is the y-intercept. Its maximum height happens at 45 m high after 2 s; this is the vertex. It hits the ground (height 0, the x-axis) after 5 s. So there is a zero at $x = 5$.



c) The graph crosses the axis at –1.
d) From the graph, the zeros occur at $x = -1$ and 5.
e) The vertex is the point where the ball is at its maximum height. The x-value is the time and the y-value is the height. The vertex is (2, 45).
f) The equation can be determined using the zeros and the vertex to solve for a.

$$\begin{aligned} y &= a(x - r)(x - s) \\ 45 &= a(2 + 1)(2 - 5) \\ 45 &= -9a \\ a &= -5 \end{aligned}$$

The equation is $y = -5(x + 1)(x - 5)$.
g) The positive zero, (5, 0), represents the point when the ball hits the ground. The negative zero, (–1, 0), is a negative time and thus has no physical meaning.
13. a) The zeros of the graph are –250 and 50 and the vertex is (–100, 9). Use this information to determine the equation.

$$\begin{aligned} &y = a(x - r)(x - s) \\ &9 = a(-100 + 250)(-100 - 50) \\ &9 = -22\,500a \\ &a = -\frac{1}{2500} \end{aligned}$$

The equation is $y = -\frac{1}{2500}(x + 250)(x - 50)$.
b) From the vertex, the price should be decreased by \$100 to obtain the maximum revenue.
14. Let x represent the number of \$1 increases. The price of the CD is \$10 plus the number of \$1 increases, $10 + x$. The number of CDs sold is 80 minus 5 for every \$1 increase, $80 - 5x$. The equation for Ryan's revenue is the number of CDs sold multiplied by the price of each CD. $R = (10 + x)(80 - 5x)$
His maximum revenue occurs at the vertex. Since we only need the price that maximizes sales, only the x-value needs to be determined. The two zeros are $x = -10$ and $x = 16$. The axis of symmetry is

$$x = \frac{-10 + 16}{2}$$

$$x = 3$$

Therefore the price is

$$\begin{aligned} p &= 10 + x \\ &= 10 + 3 \\ &= \$13 \end{aligned}$$

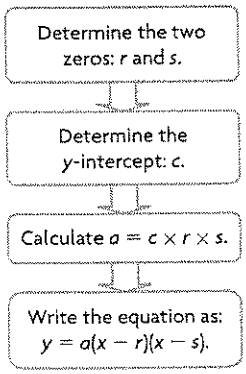
15. a) Let x represent the number of \$0.10 increases. The price is $5 - 0.10x$ dollars, and there are $700 - 10x$ packages sold. Since revenue is quantity times price, $y = (5 + 0.1x)(700 - 10x)$.
b) The zeros of the equation are –50 and 70. Therefore the axis of symmetry is $x = (-50 + 70) \div 2 = 10$.

$$\begin{aligned} y &= (5 + 0.1x)(700 - 10x) \\ y &= (5 + 0.1(10))(700 - 10(10)) \\ y &= (5 + 1)(600) \\ y &= 3600 \end{aligned}$$

The maximum revenue is \$3600.
c) $b = 700 - 10x$

$$\begin{aligned} &= 700 - 10(10) \\ &= 600 \end{aligned}$$

600 packages will be sold.
16. Answers may vary, e.g.,



17. Expand the factored form into the standard form.
a)

$$\begin{aligned} y &= (2x - 3)(x + 4) \\ y &= 2x(x + 4) - 3(x + 4) \\ y &= 2x^2 + 8x - 3x - 12 \\ y &= 2x^2 + 5x - 12 \end{aligned}$$

This equation matches iv).
b)

$$\begin{aligned} y &= (3x + 1)(4x - 3) \\ y &= 3x(4x - 3) + 1(4x - 3) \\ y &= 12x^2 - 9x + 4x - 3 \\ y &= 12x^2 - 5x - 3 \end{aligned}$$

This equation matches i).
c)

$$\begin{aligned} y &= (3 - 2x)(4 + x) \\ y &= 3(4 + x) - 2x(4 + x) \\ y &= 12 + 3x - 8x - 2x^2 \\ y &= -2x^2 - 5x + 12 \end{aligned}$$

This equation matches ii).
d)

$$\begin{aligned} y &= (3 - 4x)(1 + 3x) \\ y &= 3(1 + 3x) - 4x(1 + 3x) \\ y &= 3 + 9x - 4x - 12x^2 \\ y &= -12x^2 + 5x + 3 \end{aligned}$$

This equation matches v).
18. With \$720, Martin can buy 48 m of fence. Let m represent the length of the fence opposite his house, and n represent the length of each fence adjacent to his house. We know $m + 2n = 48$, or $m = 48 - 2n$. So, the area of the play area will be $A = mn = (48 - 2n)n$. The maximum value of A occurs at its vertex. The zeros are at 0 and 24, so the axis of symmetry is $n = (0 + 24) \div 2 = 12$. Thus, $m = 48 - 2(12) = 24$. So the dimensions that maximize the area are 12 m by 24 m.

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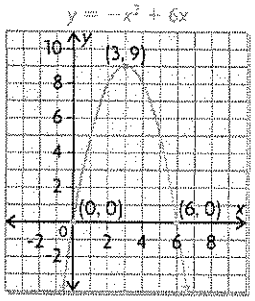
1. a) Yes. The graph is quadratic since it has a U-shape.
b) Yes, since the degree of the equation is 2.
c) No. The second differences are not a non-zero constant.

2. a) As the x-values increase, the y-values initially decrease and then increase, so this must open upward.
b) As the x-values increase, the y-values initially increase and then decrease, so this must open downward.

3.



Reading from the graph:
a) The equation for the axis of symmetry is $x = 3$.
b) The coordinates of the vertex are (3, 9).
c) The y-intercept is 0.
d) The zeros are 0 and 6.

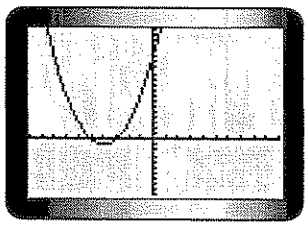
4. Because these points share the same y coordinate, they are equal distance from the axis of symmetry, so they can be used to determine the axis.

$$x = \frac{-3 + 9}{2}$$

$x = 3$

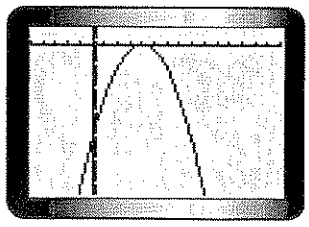
The equation for the axis of symmetry is $x = 3$.

5. a)



Reading from the graph:
y-intercept: 15
zeros: –3, –5
axis of symmetry: $x = -4$
vertex: (–4, –1)

b)



Reading from the graph:
y-intercept: –32; zeros: 4
axis of symmetry: $x = 4$; vertex: (4, 0)

6. a) To determine the height of the ball when it was kicked, let $t = 0$.

$h = -5t^2 + 15t + 0.5$
$= -5(0) + 15(0) + 0.5$
$= 0.5$

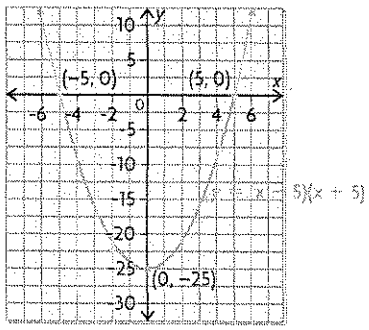
The ball was 0.5 m above the ground when it was kicked.

b) Using a table of values:

x	y
0	$h = -5(0)^2 + 15(0) + 0.5 = 0.5$
1	$h = -5(1)^2 + 15(1) + 0.5 = 10.5$
2	$h = -5(2)^2 + 15(2) + 0.5 = 10.5$
3	$h = -5(3)^2 + 15(3) + 0.5 = 0.5$
4	$h = -5(4)^2 + 15(4) + 0.5 = -19.5$

The ball will hit the ground at about 3 s.
c) From the table of values, the ball will reach its maximum height between 1 and 2 s, when $t = 1.5$ s.
d) The maximum height is
$h = -5(1.5)^2 + 15(1.5) + 0.5$
$= -11.25 + 22.5 + 0.5$
$= 11.75$
The maximum height is 11.75 m.
e) From the table of values, the height is 0.5 m. Since the ball had already reached the maximum height at 1.5 s, it will be traveling downwards.
f) The ball is 10 m off the ground just before 1 s and just after 2 s, so about at 0.9 s and 2.1 s.

7. a) y-intercept: Determine the y-value when $x = 0$.
$y = (x - 5)(x + 5)$
$y = (0 - 5)(0 + 5)$
$y = -25$
The y-intercept is –25.
Zeros: Set each factored component equal to zero.
$x - 5 = 0$ $\qquad$ $x + 5 = 0$
$x = 5$ $\qquad$ $x = -5$
The zeros are –5 and 5.
The equation of the axis of symmetry is the x-value midway between the zeros.
$x = (-5 + 5) \div 2$
$x = 0$
The axis of symmetry is $x = 0$.
Vertex: Use the axis of symmetry to solve for the y-value.
$y = (x - 5)(x + 5)$
$y = (0 - 5)(0 + 5)$
$y = -25$
The vertex is (0, –25).



b) y-intercept: Determine the y-value when $x = 0$.

$y = -(x - 6)(x - 2)$
$y = -(0 - 6)(0 - 2)$
$y = -12$

The y-intercept is -12.

Zeros: Set each factored component equal to zero.

$x - 2 = 0$ $\qquad$ $x - 6 = 0$
$x = 2$ $\qquad$ $x = 6$

The zeros are 2 and 6.

The equation of the axis of symmetry s the x-value midway between the zeros.

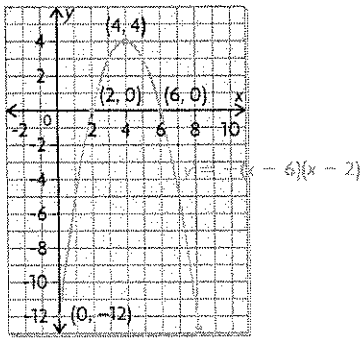
$x = (6 + 2) \div 2$
$x = 4$

The axis of symmetry is $x = 4$.

Vertex: Using the axis of symmetry, solve for the y-value.

$y = -(x - 6)(x - 2)$
$y = -(4 - 6)(4 - 2)$
$y = 4$

The vertex is (4, 4).



c) y-intercept: Determine the y-value when $x = 0$.

$y = 2(x - 1)(x + 3)$
$y = 2(0 - 1)(0 + 3)$
$y = -6$

The y-intercept is -6.

Zeros: Set each factored component equal to zero.

$x - 1 = 0$ $\qquad$ $x + 3 = 0$
$x = 1$ $\qquad$ $x = -3$

The zeros are 1 and -3.

The equation of the axis of symmetry is the x-value midway between the zeros.

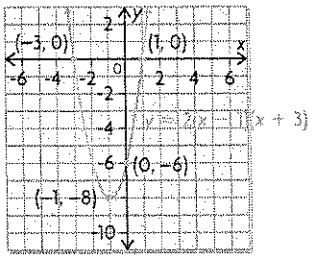
$x = (-3 + 1) \div 2$
$x = -1$

The axis of symmetry is $x = -1$.

Vertex: Using the axis of symmetry solve for the y-value.

$y = 2(x - 1)(x + 3)$
$y = 2(-1 - 1)(-1 + 3)$
$y = -8$

The vertex is $(-1, -8)$.



d) y-intercept: Determine the y-value when $x = 0$.

$y = -0.5(x + 4)^2$
$y = -0.5(0 + 4)^2$
$y = -8$

The y-intercept is -8.

Zeros: Set each factored component equal to zero.

$x + 4 = 0$
$x = -4$

The zero is -4.

The equation of the axis of symmetry is the x-value midway between the zeros, or, in this case, since there is only one zero, it is at the zero:

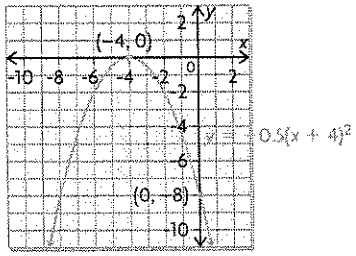
$x = -4$

The axis of symmetry is $x = -4$.

Vertex: Using the axis of symmetry, solve for the y-value.

$y = -0.5(x + 4)^2$
$y = -0.5(-4 + 4)^2$
$y = 0$

The vertex is $(-4, 0)$.



8. a) Substitute the zeros into the general form of the factored equation. Use the y-intercept to solve for a.

$y = a(x - r)(x - s)$
$y = a(x + 10)(x - 30)$
$50 = a(0 + 10)(0 - 30)$
$50 = -300a$

$$a = -\frac{1}{6}$$

The equation is $y = -\frac{1}{6}(x + 10)(x - 30)$.

b) The zeros are -10 and 30, so the axis of symmetry is

$$x = \frac{-10 + 30}{2}$$

$$x = \frac{20}{2}$$

$x = 10$

Substitute $x = 10$ into the equation and solve for y.

$$y = -\frac{1}{6}(x + 10)(x - 30)$$

$$y = -\frac{1}{6}(10 + 10)(10 - 30)$$

$$y = -\frac{1}{6}(-400)$$

$$y = \frac{200}{3}$$

Therefore the vertex is $\left(10, \frac{200}{3}\right)$.

9. From the graph the vertex is (–4, 8) and the zeros are –2 and –6. Using this information, I can solve for a.

$y = a(x - r)(x - s)$
$y = a(x + 2)(x + 6)$
$8 = a(-4 + 2)(-4 + 6)$
$8 = -4a$
$a = -2$

Therefore, the equation is $y = -2(x + 2)(x + 6)$.

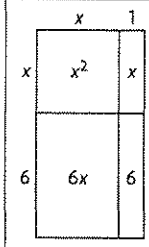
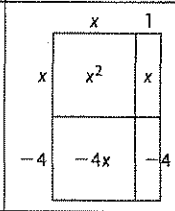
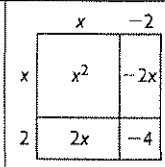
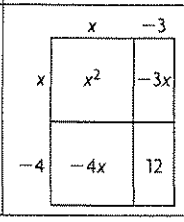
10. Answers may vary, e.g., $y = x^2$, $y = (x + 5)(x + 5)$. A parabola that has only one zero will have the same vertex and x-intercept.

Lesson 3.4: Expanding Quadratic Expressions, pages 161–168

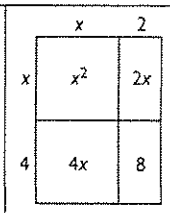
1. a) Summing like terms, the rectangle has a width of x + 1, a length of $x + 5$, and an area of $x^2 + 6x + 5$. Therefore, $(x + 5)(x + 1) = x^2 + 6x + 5$.

b) Summing like terms, the rectangle has a width of $x - 2$ a length of $x - 2$, and an area of $x^2 - 4x + 4$. Therefore, $(x - 2)(x - 2) = x^2 - 4x + 4$.

2. Place the terms of each binomial in the expression along the lengths and widths of the area diagram. Multiply the individual terms along the length and width to determine the area. Sum like terms in the area to calculate the simplified product.

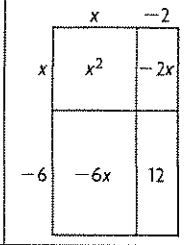
	Expression	Area Diagram	Expanded and Simplified Form
a)	$(x + 1)(x + 6)$		$x^2 + 7x + 6$
b)	$(x + 1)(x - 4)$		$x^2 - 3x - 4$
c)	$(x - 2)(x + 2)$		$x^2 - 4$
d)	$(x - 3)(x - 4)$		$x^2 - 7x + 12$

e) $(x + 2)(x + 4)$



$x^2 + 6x + 8$

f) $(x - 2)(x - 6)$



$x^2 - 8x + 12$

3. a) $(m + 3)(m + 2) = m^2 + 3m + 2m + 6$
The missing terms are m^2 and 6.

b) $(k - 2)(k + 1) = k^2 + k - 2k - 2$
The missing terms are k^2 and k.

c) $(r + 4)(r - 3) = r^2 - 3r + 4r - 12$
The missing terms are $4r$ and 12.

d) $(x - 5)(x - 2) = x^2 - 5x - 2x + 10$
The missing terms are $5x$ and $2x$.

e) $(2n + 1)(3n - 2) = 6n^2 - 4n + 3n - 2$
The missing terms are $6n^2$ and $4n$.

f) $(5m - 2)(m - 3) = 5m^2 - 15m - 2m + 6$
The missing terms are $15m$ and 6.

4. a) $(x + 2)(x + 5) = x^2 + 2x + 5x + 10$
$= x^2 + 7x + 10$

b) $(x + 2)(x + 1) = x^2 + 2x + x + 2$
$= x^2 + 3x + 2$

c) $(x + 2)(x - 3) = x^2 + 2x - 3x - 6$
$= x^2 - x - 6$

d) $(x + 2)(x - 1) = x^2 + 2x - x - 2$
$= x^2 + x - 2$

e) $(x - 4)(x - 2) = x^2 - 4x - 2x + 8$
$= x^2 - 6x + 8$

f) $(x - 5)(x - 3) = x^2 - 5x - 3x + 15$
$= x^2 - 8x + 15$

5. a) $(5x + 2)(x + 2) = 5x^2 + 2x + 10x + 4$
$= 5x^2 + 12x + 4$

b) $(x + 2)(4x + 1) = 4x^2 + 8x + x + 2$
$= 4x^2 + 9x + 2$

c) $(x - 2)(7x + 3) = 7x^2 - 14x + 3x - 6$
$= 7x^2 - 11x - 6$

d) $(3x - 2)(x + 1) = 3x^2 - 2x + 3x - 2$
$= 3x^2 + x - 2$

e) $(x - 2)(4x - 6) = 4x^2 - 8x - 6x + 12$
$= 4x^2 - 14x + 12$

f) $(7x - 5)(x - 3) = 7x^2 - 5x - 21x + 15$
$= 7x^2 - 26x + 15$

6. a) $(x + 3)(x - 3) = x^2 + 3x - 3x - 9$
$= x^2 - 9$

b) $(x + 6)(x - 6) = x^2 + 6x - 6x - 36$
$= x^2 - 36$

c) $(2x - 1)(2x + 1) = 4x^2 - 2x + 2x - 1$
$= 4x^2 - 1$

d) $(3x - 3)(3x + 3) = 9x^2 - 9x + 9x - 9$
$= 9x^2 - 9$

e) $(4x-6)(4x+6) = 16x^2 - 24x + 24x - 36$
$= 16x^2 - 36$

f) $(7x-5)(7x+5) = 49x^2 - 35x + 35x - 25$
$= 49x^2 - 25$

7. a) $(x+1)^2 = (x+1)(x+1)$
$= x^2 + x + x + 1$
$= x^2 + 2x + 1$

b) $(a+4)^2 = (a+4)(a+4)$
$= a^2 + 4a + 4a + 16$
$= a^2 + 8a + 16$

c) $(c-1)^2 = (c-1)(c-1)$
$= c^2 - c - c + 1$
$= c^2 - 2c + 1$

d) $(5y-2)^2 = (5y-2)(5y-2)$
$= 25y^2 - 10y - 10y + 4$
$= 25y^2 - 20y + 4$

e) $(6z-5)^2 = (6z-5)(6z-5)$
$= 36z^2 - 30z - 30z + 25$
$= 36z^2 - 60z + 25$

f) $(-3d+5)^2 = (-3d+5)(-3d+5)$
$= 9d^2 - 15d - 15d + 25$
$= 9d^2 - 30d + 25$

8. a) $A = lw$
$= (2m+3)(4m-4)$
$= 8m^2 + 12m - 8m - 12$
$= 8m^2 + 4m - 12$

b) $A = s^2$
$= (3m+2)^2$
$= (3m+2)(3m+2)$
$= 9m^2 + 6m + 6m + 4$
$= 9m^2 + 12m + 4$

c) $A = bh$
$= (2x-4)(5x+3)$
$= 10x^2 - 20x + 6x - 12$
$= 10x^2 - 14x - 12$

d) $A = \frac{b+t}{2}h$

$A = \frac{(3x+1)+(x+1)}{2}(2x+2)$

$A = \frac{4x+2}{2}(2x+2)$

$A = (2x+1)(2x+2)$

$A = 4x^2 + 2x + 4x + 2$

$A = 4x^2 + 6x + 2$

9. a) $4(x-6)(x+7) = 4(x^2 - 6x + 7x - 42)$
$= 4(x^2 + x - 42)$
$= 4x^2 + 4x - 168$

b) $-(x+3)(4x-1) = -(4x^2 + 12x - x - 3)$
$= -(4x^2 + 11x - 3)$
$= -4x^2 - 11x + 3$

c) $6x(x+1)^2 = 6x(x+1)(x+1)$
$= 6x(x^2 + x + x + 1)$
$= 6x(x^2 + 2x + 1)$
$= 6x^3 + 12x^2 + 6x$

d) $(x+4)(x-2) + (x-1)(x+5)$
$= (x^2 + 4x - 2x - 8) + (x^2 - x + 5x - 5)$
$= x^2 + 2x - 8 + x^2 + 4x - 5$
$= 2x^2 + 6x - 13$

e) $(4x-1)(4x+1) - (x+3)^2$
$= (4x-1)(4x+1) - (x+3)(x+3)$
$= (16x^2 - 4x + 4x - 1) - (x^2 + 3x + 3x + 9)$
$= 16x^2 - 1 - (x^2 + 6x + 9)$
$= 15x^2 - 6x - 10$

f) $2(3x+4)^2 - 3(x-2)^2$
$= 2(3x+4)(3x+4) - 3(x-2)(x-2)$
$= 2(9x^2 + 12x + 12x + 16) - 3(x^2 - 2x - 2x + 4)$
$= 2(9x^2 + 24x + 16) - 3(x^2 - 4x + 4)$
$= 18x^2 + 48x + 32 - 3x^2 + 12x - 12$
$= 15x^2 + 60x + 20$

10. a) $(x+y)(2x+3y) = 2x^2 + 2xy + 3xy + 3y^2$
$= 2x^2 + 5xy + 3y^2$

b) $(x+2y)(3x+y) = 3x^2 + 6xy + xy + 2y^2$
$= 3x^2 + 7xy + 2y^2$

c) $(3x-2y)(5x+4y) = 15x^2 - 10xy + 12xy - 8y^2$
$= 15x^2 + 2xy - 8y^2$

d) $(8x-y)(7x+2y) = 56x^2 - 7xy + 16xy - 2y^2$
$= 56x^2 + 9xy - 2y^2$

e) $(6x-5y)(6x+5y) = 36x^2 - 30xy + 30xy - 25y^2$
$= 36x^2 - 25y^2$

f) $(9x-7y)^2 = (9x-7y)(9x-7y)$
$= 81x^2 - 63xy - 63xy + 49y^2$
$= 81x^2 - 126xy + 49y^2$

11. a) From the graph, this parabola has zeros at $x = -2$ and $x = 4$. Therefore, its equation looks like $y = a(x+2)(x-4)$ in factored form. Also, the graph has a y-intercept at $(0, -8)$. Substitute the coordinates of this point into the equation to solve for a.

$y = a(x+2)(x-4)$
$-8 = a(0+2)(0-4)$
$-8 = a(2)(-4)$
$-8a = -8$
$a = 1$

The equation is $y = (x+2)(x-4)$ in factored form.
Expand to express the equation in standard form.

$y = (x+2)(x-4)$
$y = x^2 + 2x - 4x - 8$
$y = x^2 - 2x - 8$

The equation is $y = x^2 - 2x - 8$ in standard form.

b) From the graph, this parabola has zeros at $x = -2$ and $x = -4$. Therefore, its equation looks like $y = a(x+2)(x+4)$ in factored form. Also, the graph has a y-intercept at $(0, -8)$. Substitute the coordinates of this point into the equation to solve for a.

$y = a(x+2)(x+4)$
$-8 = a(0+2)(0+4)$
$-8 = a(2)(4)$
$8a = -8$
$a = -1$

The equation is $y = -(x+2)(x+4)$ in factored form.
Expand to express the equation in standard form.

$y = -(x+2)(x+4)$
$y = -(x^2 + 2x + 4x + 8)$
$y = -(x^2 + 6x + 8)$
$y = -x^2 - 6x - 8$

The equation is $y = -x^2 - 6x - 8$ in standard form.

c) From the graph, this parabola has zeros at $x = 0$ and $x = 4$. Therefore, its equation looks like $y = ax(x - 4)$ in factored form. Also, the graph has a vertex at $(2, -8)$. Substitute the coordinates of this point into the equation to solve for a.

$$\begin{aligned} y &= ax(x-4) \\ -8 &= a(2)(2-4) \\ -8 &= a(2)(-2) \\ -4a &= -8 \\ a &= 2 \end{aligned}$$

The equation is $y = 2x(x - 4)$ in factored form. Expand to express the equation in standard form.

$$\begin{aligned} y &= 2x(x-4) \\ y &= 2x^2 - 8x \end{aligned}$$

The equation is $y = 2x^2 - 8x$ in standard form.

d) From the graph, this parabola has zeros at $x = 2$ and $x = 6$. Therefore, its equation looks like $y = a(x - 2)(x - 6)$ in factored form. Also, the graph has a y-intercept at $(0, -6)$.

Substitute the coordinates of this point into the equation to solve for a.

$$\begin{aligned} y &= a(x-2)(x-6) \\ -6 &= a(0-2)(0-6) \\ -6 &= a(-2)(-6) \\ 12a &= -6 \\ a &= -0.5 \end{aligned}$$

The equation is $y = -0.5(x - 2)(x - 6)$ in factored form. Expand to express the equation in standard form.

$$\begin{aligned} y &= -0.5(x-2)(x-6) \\ y &= -0.5(x^2 - 2x - 6x + 12) \\ y &= -0.5(x^2 - 8x + 12) \\ y &= -0.5x^2 + 4x - 6 \end{aligned}$$

The equation is $y = -0.5x^2 + 4x - 6$ in standard form.

12. a) Inserting the zeros, the equation looks like $y = a(x + 1)(x - 7)$ in factored form. Substitute the point $(3, 5)$ into the equation to solve for a.

$$\begin{aligned} y &= a(x+1)(x-7) \\ 5 &= a(3+1)(3-7) \\ 5 &= a(4)(-4) \\ -16a &= 5 \end{aligned}$$

$$a = -\frac{5}{16}$$

The equation is $y = -\frac{5}{16}(x+1)(x-7)$ in factored form.

Expand to put this equation into standard form.

$$y = -\frac{5}{16}(x+1)(x-7)$$

$$y = -\frac{5}{16}(x^2 + x - 7x - 7)$$

$$y = -\frac{5}{16}(x^2 - 6x - 7)$$

$$y = -\frac{5}{16}x^2 + \frac{15}{8}x + \frac{35}{16}$$

The equation is $y = -\frac{5}{16}x^2 + \frac{15}{8}x + \frac{35}{16}$ in standard form. The parabola opens downward, since $a < 0$.

b) Inserting the zeros, the equation looks like $y = a(x + 1)(x + 5)$ in factored form. Substitute the point $(-3, -4)$ into the equation to solve for a.

$$\begin{aligned} y &= a(x+1)(x+5) \\ -4 &= a(-3+1)(-3+5) \\ -4 &= a(-2)(2) \\ -4a &= -4 \\ a &= 1 \end{aligned}$$

The equation is $y = (x + 1)(x + 5)$ in factored form. Expand to put this equation into standard form.

$$\begin{aligned} y &= (x+1)(x+5) \\ y &= (x^2 + x + 5x + 5) \\ y &= x^2 + 6x + 5 \end{aligned}$$

The equation is $y = x^2 + 6x + 5$ in standard form. The parabola opens upward, since $a > 0$.

c) Inserting the zeros, the equation looks like $y = a(x - 3)(x - 7)$ in factored form. Substitute the point $(0, 3)$ into the equation to solve for a.

$$\begin{aligned} y &= a(x-3)(x-7) \\ 3 &= a(0-3)(0-7) \\ 3 &= a(-3)(-7) \\ 21a &= 3 \end{aligned}$$

$$a = \frac{1}{7}$$

The equation is $y = \frac{1}{7}(x-3)(x-7)$ in factored form.

Expand to put this equation into standard form.

$$y = \frac{1}{7}(x-3)(x-7)$$

$$y = \frac{1}{7}(x^2 - 3x - 7x + 21)$$

$$y = \frac{1}{7}(x^2 - 10x + 21)$$

$$y = \frac{1}{7}x^2 - \frac{10}{7}x + 3$$

The equation is $y = \frac{1}{7}x^2 - \frac{10}{7}x + 3$ in standard form.

The parabola opens upward, since $a > 0$.

d) Inserting the zeros, the equation looks like $y = a(x + 2)(x - 6)$ in factored form. Substitute the point $(-1, -1)$ into the equation to solve for a.

$$\begin{aligned} y &= a(x+2)(x-6) \\ (-1) &= a(-1+2)(-1-6) \\ -1 &= a(1)(-7) \\ -7a &= -1 \end{aligned}$$

$$a = \frac{1}{7}$$

The equation is $y = \frac{1}{7}(x+2)(x-6)$ in factored form.

Expand to put this equation into standard form.

$$y=\frac{1}{7}(x+2)(x-6)$$

$$y=\frac{1}{7}(x^2+2x-6x-12)$$

$$y=\frac{1}{7}(x^2-4x-12)$$

$$y=\frac{1}{7}x^2-\frac{4}{7}x-\frac{12}{7}$$

The equation is $y=\frac{1}{7}x^2-\frac{4}{7}x-\frac{12}{7}$ in standard form.

The parabola opens upward, since $a > 0$.

e) Inserting the zeros, the equation looks like $y = a(x + 2)(x - 8)$ in factored form. Substitute the point (3, 7) into the equation to solve for a.

$$y = a(x+2)(x-8)$$
$$7 = a(3+2)(3-8)$$
$$7 = a(5)(-5)$$
$$-25a = 7$$
$$a=-\frac{7}{25}$$

The equation is $y=-\frac{7}{25}(x+2)(x-8)$ in factored form.

Expand to put this equation into standard form.

$$y=-\frac{7}{25}(x+2)(x-8)$$

$$y=-\frac{7}{25}(x^2+2x-8x-16)$$

$$y=-\frac{7}{25}(x^2-6x-16)$$

$$y=-\frac{7}{25}x^2+\frac{42}{25}x+\frac{112}{25}$$

The equation is $y=-\frac{7}{25}x^2+\frac{42}{25}x+\frac{112}{25}$ in standard form. The parabola opens downward, since $a < 0$.

13. Agree with Bill. Check the areas formed by both sets of dimensions.

$$A = (2x + 4)(x + 5)$$
$$= 2x^2 + 4x + 10x + 20$$
$$= 2x^2 + 14x + 20$$

Therefore, the rectangle could be $2x + 4$ by $x + 5$, since these dimensions form the required area.

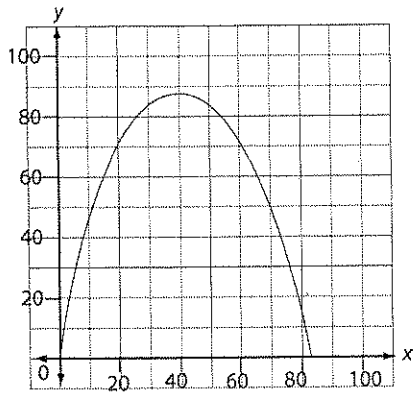
$$A = (2x + 10)(x + 2)$$
$$= 2x^2 + 10x + 4x + 20$$
$$= 2x^2 + 14x + 20$$

Therefore, the rectangle could also be $2x + 10$ by $x + 2$, since these dimensions also form the required area. Observe that the expressions for the area of either rectangle in factored form are related by a factor of 2.

$$(2x + 4)(x + 5) = 2(x + 2)(x + 5)$$
$$= (2x + 10)(x + 2)$$

14. Both binomials in the expression have 1 as the largest exponent on any term. Therefore, their product will have 2 as its largest exponent on any term, making the product quadratic.

15. First, sketch the arch on a coordinate plane. Let the origin lie where the left side of the arch reaches the ground.



The arch has zeros at $x = 0$ and at $x = 84$; so, it has an equation that looks like $y = ax(x - 84)$ in factored form. The equation for its axis of symmetry is:

$$x = (0 + 84) \div 2$$
$$x = 42$$

So, it has a vertex at (42, 88). Substitute the coordinates of the vertex to solve for a.

$$y = ax(x-84)$$
$$88 = a(42)(42-84)$$
$$88 = a(42)(-42)$$
$$-1764a = 88$$
$$a=-\frac{88}{1764}$$

The arch has the equation $y=-\frac{88}{1764}x(x-84)$ in factored form. Expand to put the equation into standard form.

$$y=-\frac{88}{1764}x(x-84)$$

$$y=-\frac{88}{1764}x^2+\frac{176}{42}x$$

The arch has the equation $y=-\frac{88}{1764}x^2+\frac{176}{42}x$ in standard form.

16. Jay is incorrect. For example, sometimes the like terms sum to zero, creating a binomial. For example:

$$(x + 2)(x - 2) = x^2 + 2x - 2x - 4$$
$$= x^2 - 4$$

and:

$$(3x + y)(3x - y) = 9x^2 + 3xy - 3xy - y^2$$
$$= 9x^2 - y^2$$

17. a)

$$(x + 3)^3 = (x + 3)(x + 3)(x + 3)$$
$$= (x^2 + 3x + 3x + 9)(x + 3)$$
$$= (x^2 + 6x + 9)(x + 3)$$
$$= x(x^2 + 6x + 9) + 3(x^2 + 6x + 9)$$
$$= x^3 + 6x^2 + 9x + 3x^2 + 18x + 27$$
$$= x^3 + 9x^2 + 27x + 27$$

b) $(2x-2)^3 = (2x-2)(2x-2)(2x-2)$
$= (2x-2)(4x^2-8x+4)$
$= 2x(4x^2-8x+4) - 2(4x^2-8x+4)$
$= 8x^3 - 16x^2 + 8x - 8x^2 + 16x - 8$
$= 8x^3 - 24x^2 + 24x - 8$

c) $(4x+2y)^3 = (4x+2y)(4x+2y)(4x+2y)$
$= (4x+2y)(16x^2+8xy+8xy+4y^2)$
$= (4x+2y)(16x^2+16xy+4y^2)$
$= 4x(16x^2+16xy+4y^2)$
$+ 2y(16x^2+16xy+4y^2)$
$= 64x^3 + 64x^2y + 16xy^2$
$+ 32x^2y + 32xy^2 + 8y^3$
$= 64x^3 + 96x^2y + 48xy^2 + 8y^3$

d) $[(x+2)(x-2)]^2 = (x^2+2x-2x-4)^2$
$= (x^2-4)^2$
$= (x^2-4)(x^2-4)$
$= x^4 - 4x^2 - 4x^2 + 16$
$= x^4 - 8x^2 + 16$

e) $(x+6)(x+3)(x-6)(x-3)$
$= [(x+6)(x-6)][(x+3)(x-3)]$
$= [x^2+6x-6x-36][x^2+3x-3x-9]$
$= [x^2-36][x^2-9]$
$= x^4 - 36x^2 - 9x^2 + 324$
$= x^4 - 45x^2 + 324$

f) $(3x^2+6x-1)^2$
$= (3x^2+6x-1)(3x^2+6x-1)$
$= 3x^2(3x^2+6x-1) + 6x(3x^2+6x-1) - (3x^2+6x-1)$
$= 9x^4 + 18x^3 - 3x^2 + 18x^3 + 36x^2 - 6x - 3x^2 - 6x + 1$
$= 9x^4 + 18x^3 + 18x^3 - 3x^2 + 36x^2 - 3x^2 - 6x - 6x + 1$
$= 9x^4 + 36x^3 + 30x^2 - 12x + 1$

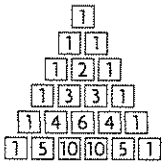
18. a) $(a+b)^1 = a+b$

b) $(a+b)^2 = (a+b)(a+b)$
$= a^2 + ab + ab + b^2$
$= a^2 + 2ab + b^2$

c) $(a+b)^3 = (a+b)(a+b)^2$
$= (a+b)(a^2+2ab+b^2)$
$= a(a^2+2ab+b^2) + b(a^2+2ab+b^2)$
$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$
$= a^3 + 2a^2b + a^2b + ab^2 + 2ab^2 + b^3$
$= a^3 + 3a^2b + 3ab^2 + b^3$

d) $(a+b)^4$
$= (a+b)(a+b)^3$
$= (a+b)(a^3+3a^2b+3ab^2+b^3)$
$= a(a^3+3a^2b+3ab^2+b^3) + b(a^3+3a^2b+3ab^2+b^3)$
$= a^4 + 3a^3b + 3a^2b^2 + ab^3 + a^3b + 3a^2b^2 + 3ab^3 + b^4$
$= a^4 + 3a^3b + a^3b + 3a^2b^2 + 3a^2b^2 + ab^3 + 3ab^3 + b^4$
$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

19. For example, I noticed that the sum of the exponents in each term always add to the exponent on the factored form (e.g., in term $4a^3b$, the exponents of a and b sum to 4, the exponent of the factored form). Also, I noticed that when the terms are arranged so that a decreases in degree to zero, the coefficients of each term make a symmetrical pattern (e.g., in the last line, the coefficients are 1, 4, 6, 4, 1). Finally, I noticed that the coefficients can be predicted using the following pyramid, where each number is the sum of the two numbers above it.



Lesson 3.5: Quadratic Models Using Factored Form, pages 169–178

1. a) For example, it looks as though a parabola could pass through all points on the graph. It would intersect the x-axis at $x = 2$ and $x = 4$. In factored form, the parabola would have equation $y = a(x-2)(x-4)$. Substitute a known point, such as the y-intercept, into the equation to solve for a.

$y = a(x-2)(x-4)$
$-2 = a(0-2)(0-4)$
$-2 = 8a$
$a = -0.25$

The curve of best fit is $y = -0.25(x-2)(x-4)$ in factored form. Expand this equation to convert to standard form.

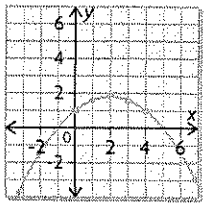
$y = -0.25(x-2)(x-4)$
$y = -0.25(x^2-2x-4x+8)$
$y = -0.25(x^2-6x+8)$
$y = -0.25x^2 + 1.5x - 2$

b) Substitute $x = 1$ into the equation for the curve of good fit to estimate y.

$y = -0.25x^2 + 1.5x - 2$
$y = -0.25(1)^2 + 1.5(1) - 2$
$y = -0.25 + 1.5 - 2$
$y = -0.75$

The value of y is about -0.75.

2. a) For example, this graph shows a curve of good fit.



The curve of good fit has zeros at $x = -1$ and $x = 5$. Its equation is $y = a(x+1)(x-5)$ in factored form. The y-intercept occurs at $(0, 1)$; substitute this point into the equation to solve for a.

$y = a(x+1)(x-5)$
$1 = a(0+1)(0-5)$
$1 = a(1)(-5)$
$1 = -5a$
$a = -0.2$

The curve has the equation $y = -0.2(x+1)(x-5)$ in factored form. Expand to put it in standard form.

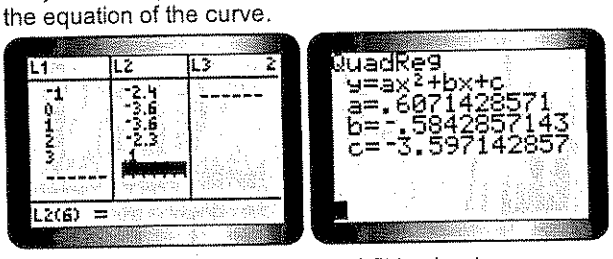
$y = -0.2(x+1)(x-5)$
$y = -0.2(x^2+x-5x-5)$
$y = -0.2(x^2-4x-5)$
$y = -0.2x^2 + 0.8x + 1$

b) Using the equation in part a), solve for y when $x = 1.5$.

$$y = -0.2x^2 + 0.8x + 1$$
$$y = -0.2(1.5)^2 + 0.8(1.5) + 1$$
$$y = -0.45 + 1.2 + 1$$
$$y = 1.75$$

The ball is about 1.75 m above the ground after it has travelled 1.5 m horizontally.

3. a) For example, use graphing technology to determine the equation of the curve.



The equation of the curve of good fit is about $y = 0.6x^2 - 0.6x - 3.6$.

b) For example, solve for y when $x = 3.2$.

$$y = 0.6x^2 - 0.6x - 3.6$$
$$y = 0.6(3.2)^2 - 0.6(3.2) - 3.6$$
$$y = 6.144 - 1.92 - 3.6$$
$$y = 0.624$$

The value of y is about 0.6 when $x = 3.2$.

4. a) For example, the parabola has zeros at $x = 1$ and $x = -3$. Its equation is $y = a(x - 1)(x + 3)$ in factored form. Substitute the coordinates of another known point, such as the y-intercept, into this equation to solve for a.

$$y = a(x - 1)(x + 3)$$
$$-6 = a(0 - 1)(0 + 3)$$
$$-6 = -3a$$
$$a = 2$$

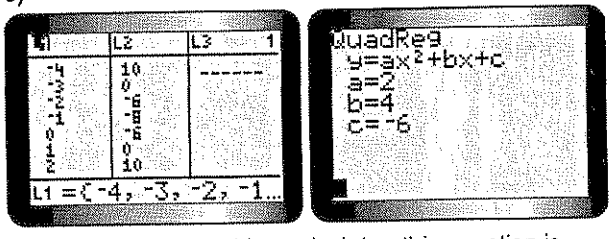
The equation is $y = 2(x - 1)(x + 3)$ in factored form.

b) For example, expand the equation above to change it to standard form.

$$y = 2(x - 1)(x + 3)$$
$$y = 2(x^2 - x + 3x - 3)$$
$$y = 2(x^2 + 2x - 3)$$
$$y = 2x^2 + 4x - 6$$

The equation is $y = 2x^2 + 4x - 6$ in standard form.

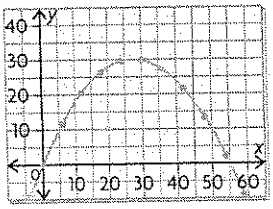
c)



According to the graphing calculator, this equation is exactly correct.

5. Answers may vary, e.g.,

a)



b) The curve of good fit has zeros at $x = 0$ and $x = 55$. Its equation looks like $y = ax(x - 55)$ in factored form. The curve also has a vertex at about (27.5, 30). Substitute these coordinates into the equation to solve for a.

$$y = ax(x - 55)$$
$$30 = a(27.5)(27.5 - 55)$$
$$30 = a(27.5)(-27.5)$$
$$30 \doteq -756a$$
$$a \doteq -\frac{5}{126}$$

The curve of good fit has equation $y = -\frac{5}{126}x(x - 55)$.

c) From the graph, the balloon appears to have reached its maximum height after travelling a horizontal distance of 27.5 m. Substitute this value into the equation to find the maximum height.

$$y = -\frac{5}{126}x(x - 55)$$
$$y = -\frac{5}{126}(27.5)[(27.5) - 55]$$
$$y = -\frac{5}{126}(27.5)(-27.5)$$
$$y = -\frac{5}{126}(-756)$$
$$y = 30$$

The maximum height of the balloon was 30 m.

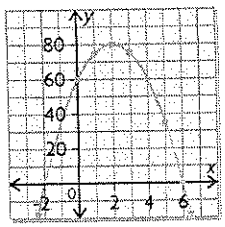
d) Substitute the horizontal distance into the equation to solve for the height.

$$y = -\frac{5}{126}x(x - 55)$$
$$y = -\frac{5}{126}(40)(40 - 55)$$
$$y = -\frac{5}{126}(40)(-15)$$
$$y = -\frac{5}{126}(-600)$$
$$y = 23.8$$

Therefore, the balloon was at a height of about 24 m after travelling a horizontal distance of 40 m.

6. a) Since the flare was shot from the top of the building at time zero, from the table of values, the height of the building is 60 m.

b) For example, the graph below shows the data and a curve of good fit.



c) For example, the curve of good fit has zeros at $x = -2$ and at $x = 6$; so, it is $y = a(x - 6)(x + 2)$ in factored form. The graph also has a y-intercept at $y = 60$; so, substitute the coordinates of this point into the equation to solve for a.

$$\begin{aligned} y &= a(x - 6)(x + 2) \\ 60 &= a(0 - 6)(0 + 2) \\ 60 &= a(-6)(2) \\ 60 &= -12a \\ a &= -5 \end{aligned}$$

This curve is $y = -5(x - 6)(x + 2)$.

d) For example,

$$\begin{aligned} y &= -5(x - 6)(x + 2) \\ y &= -5(2.5 - 6)(2.5 + 2) \\ y &= -5(-3.5)(4.5) \\ y &= 78.75 \end{aligned}$$

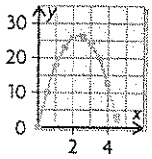
The flare is 79 m high after 2.5 s.

7. a) The height of the glider is the same as the height of the platform at time zero; so, the platform is 10 m tall.

b) For example, let x represent time and y represent height. Use graphing technology to find that the equation $y = 1.2x^2 - 12x + 10$ describes a curve of good fit.

c) For example, the table of values is mostly symmetric, with most heights being equal to another height at a different time. The centre of the pattern is –20.0 m, which is the lowest height in the table. Therefore, this height corresponds to the vertex of the quadratic relation that represents this data. As a result, the lowest height of the hang-glider was –20.0 m, which occurred after 5 s.

8. a) For example,



b) The zeros of the curve are at about $x = 0$ and $x = 4.6$.

c) The equation looks like $y = ax(x - 4.6)$ in factored form. The point (2, 26) is on the curve; so, substitute the coordinates of this point to solve for a.

$$\begin{aligned} y &= ax(x - 4.6) \\ 26 &= a(2)(2 - 4.6) \\ 26 &= -5.2a \\ a &= -5.0 \end{aligned}$$

The equation $y = -5x(x - 4.6)$ models this situation.

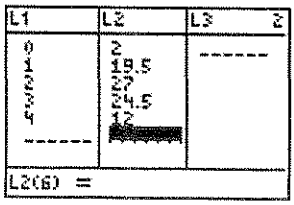
d) The vertex of the curve must occur exactly halfway between the zeros; so, it has an x-coordinate of $(0 + 4.6) \div 2 = 2.3$. Substitute this value into the equation to solve for the maximum height.

$$\begin{aligned} y &= -5x(x - 4.6) \\ y &= -5(2.3)(2.3 - 4.6) \\ y &= -5(2.3)(-2.3) \\ y &= 26.45 \end{aligned}$$

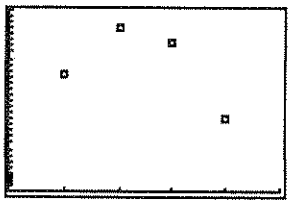
The ball has a maximum height of about 26.5 m.

9. Answers may vary. For example:

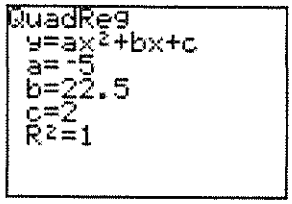
a) I entered the data into a graphing calculator.



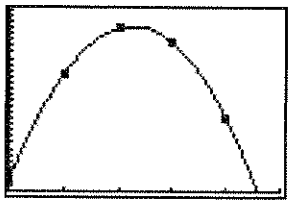
Then I plotted the data.



I used quadratic regression to determine an equation. The equation is $y = -5x^2 + 22.5x + 2$, where x is the time elapsed and y is the height.



I graphed the equation to check. It went through all the points.

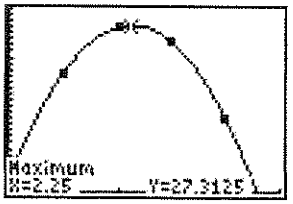


b) Substitute $x = 3.8$ into the equation and solve for y.

$$\begin{aligned} y &= -5x^2 + 22.5x + 2 \\ y &= -5(3.8)^2 + 22.5(3.8) + 2 \\ y &= -72.2 + 85.5 + 2 \\ y &= 15.3 \end{aligned}$$

The rocket was about 15.3 m high after 3.8 s.

c) I used the calculator to determine the maximum height, which is about 27.3 m at about 2.3 s.

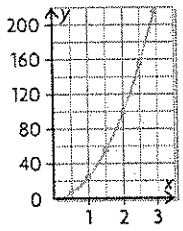


10. a)

Period (s)	Length (cm)	First Difference	Second Difference
0.5	6.2		
		18.6	
1.0	24.8		12.4
		31.0	
1.5	55.8		12.4
		43.4	
2.0	99.2		12.4
		55.8	
2.5	155.0		

Since the second differences are a constant of 12.4, this data can be represented by a quadratic relation.

b) For example:



c) For example, since the curve of good fit has a vertex at (0, 0), its equation must be of the form $y = a(x - 0)^2$, or just $y = ax^2$. The point (2.5, 155.0) is on the curve; so, substitute these coordinates to solve for a.

$$\begin{aligned} y &= ax^2 \\ 155.0 &= a(2.5)^2 \\ 155.0 &= 6.25a \\ a &= 24.8 \end{aligned}$$

This graph has an equation of $y = 24.8x^2$.

d) For example, let $y = 80$ and solve for x.

$$\begin{aligned} y &= 24.8x^2 \\ 80 &= 24.8x^2 \\ x^2 &\doteq 3.2 \\ x &\doteq 1.8 \end{aligned}$$

A pendulum 80.0 cm long has a period of about 1.8 s.

e) For example, let $x = 2.3$ and solve for y.

$$\begin{aligned} y &= 24.8x^2 \\ y &= 24.8(2.3)^2 \\ y &= 24.8(5.29) \\ y &\doteq 131.2 \end{aligned}$$

A pendulum with a period of 2.3 s is about 131.2 cm long.

11. Let n represent the number of the diagram. Diagram 1 has side lengths of 2 dots, diagram 2 has side lengths of 4 dots, and diagram 3 has side lengths of 6 dots. The pattern shows the side length of each square is $2n$. The total number of dots in any square dot pattern is equal to the number of dots on the side squared, or $(2n)^2$. There are $[2(20)]^2 = 1600$ dots in diagram 20.

12. a) & b)

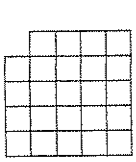


Figure 4

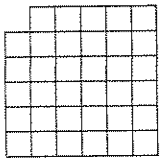


Figure 5

x	y	First Difference	Second Difference
1	3		
		5	
2	8		2
		7	
3	15		2
		9	
4	24		2
		11	
5	35		

c) Suppose another small square existed in the top-left corner of each figure. Then, each figure would be a large square. The side length of each of these large squares would be equal to one more than the figure number, or $x + 1$. Then, the area of each of these figures would be $y = (x + 1)^2$. However, I added on an imaginary square of area 1 to the corner of each figure. Therefore, the actual equation relating the figure number to the area is $y = (x + 1)^2 - 1$.

d)

x	y	First Difference	Second Difference
–4	8		
		–5	
–3	3		2
		–3	
–2	0		2
		–1	
–1	–1		2
		1	
0	0		2
		3	
1	3		2
		5	
2	8		

Therefore, zeros exist at (–2, 0) and (0, 0).

e) Check (–2, 0):

$$\begin{aligned} LS &= y \\ &= 0 \\ RS &= (x + 1)^2 - 1 \\ &= (-2 + 1)^2 - 1 \\ &= 1 - 1 \\ &= 0 \\ LS &= RS \end{aligned}$$

Check (0, 0):

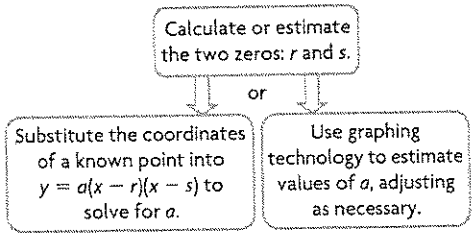
$$\begin{aligned} LS &= y \\ &= 0 \\ RS &= (x + 1)^2 - 1 \\ &= (0 + 1)^2 - 1 \\ &= 1 - 1 \\ &= 0 \\ LS &= RS \end{aligned}$$

Both of these zeros satisfy the equation; therefore, they are the zeros of the relation.

f) From part c), x represents the figure number. The smallest figure number is 1, so x must be greater than or equal to 1 and x must be an integer.

13. The factored form of a quadratic relation can only be used if the curve crosses the x-axis. In other cases, no, it cannot be used.

14. Answers may vary, e.g.,


Calculate or estimate the two zeros: r and s.
or
Substitute the coordinates of a known point into y = a(x − r)(x − s) to solve for a.
Use graphing technology to estimate values of a, adjusting as necessary.

15. a) Observe the pattern formed by the models. Models 2 and 3 look as though they could be broken into two equal square-based prism parts, if one more cube were to be added to the corner of one of these prisms.

Let y represent the number of cubes and x represent the model number. Then each of these prisms has a height of 1, and the square base has a side length equal to the model number, x. Thus, the total number of cubes in one of these square-based prisms is equal to $x \times x \times 1 = x^2$. Since there are two of these prisms in any model, the relationship becomes $y = 2x^2$. However, recall that we are pretending that one extra cube was added to the corner of one base. The actual relationship relating the model number to the number of cubes is, therefore, $y = 2x^2 - 1$. Substitute $x = 15$ into the equation to solve for the number of cubes in model 15.

$$\begin{aligned} y &= 2x^2 - 1 \\ y &= 2(15)^2 - 1 \\ y &= 450 - 1 \\ y &= 449 \end{aligned}$$

There are 449 cubes in model 15.

b) Substitute $y = 1249$ into the equation and solve for x.

$$\begin{aligned} y &= 2x^2 - 1 \\ 1249 &= 2x^2 - 1 \\ 1250 &= 2x^2 \\ x^2 &= 625 \\ x &= 25 \end{aligned}$$

(Since x must be positive, ignore the negative root.)
Model 25 can be built using 1249 cubes.

Lesson 3.6: Exploring Quadratic and Exponential Graphs, pages 179–182

1. a) $\frac{3^4}{3^4} = 3^{4-4}$

$= 3^0$

b) $\frac{3^4}{3^4} = \frac{3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3}$

$= 1$

c) I conclude that $3^0 = 1$. By extension, I conclude that any non-zero base raised to the exponent 0 equals 1.

d) $\frac{5^3}{5^3} = 5^{3-3}$

$= 5^0$

Also,

$\frac{5^3}{5^3} = \frac{5 \times 5 \times 5}{5 \times 5 \times 5}$

$= 1$

I conclude that $5^0 = 1$. By extension, conclude that any non-zero base raised to the exponent 0 equals 1.

2. a) $\frac{3^3}{3^4} = 3^{3-4}$

$= 3^{-1}$

b) $\frac{3^3}{3^4} = \frac{3 \times 3 \times 3}{3 \times 3 \times 3 \times 3}$

$= \frac{1}{3}$

c) I conclude that $3^{-1} = \frac{1}{3}$. By extension, I conclude that any non-zero base raised to the exponent -1 is equal to the reciprocal of the base.

d) $\frac{5^2}{5^4} = 5^{2-4}$

$= 5^{-2}$

Also,

$\frac{5^2}{5^4} = \frac{5 \times 5}{5 \times 5 \times 5 \times 5}$

$= \frac{1}{5^2}$

I conclude that $5^{-2} = \frac{1}{5^2}$. By extension, I conclude that any non-zero base raised to the exponent -2 is equal to the reciprocal of the base squared.

e) $\frac{5^2}{5^5} = 5^{2-5}$

$= 5^{-3}$

Also,

$\frac{5^2}{5^5} = \frac{5 \times 5}{5 \times 5 \times 5 \times 5 \times 5}$

$= \frac{1}{5^3}$

I conclude that $5^{-3} = \frac{1}{5^3}$. By extension, I conclude that any non-zero base raised to the exponent $-n$, $n > 0$, is equal to the reciprocal of the base to the exponent n.

3. a) $2^{-4} = \frac{1}{2^4}$

$= \frac{1}{16}$

b) $4^{-1} = \frac{1}{4}$

c) $8^0 = 1$

d) $5^{-2} = \frac{1}{5^2}$

$= \frac{1}{25}$

e) $3^{-4} = \frac{1}{3^4}$

$= \frac{1}{81}$

f) $7^{-2} = \frac{1}{7^2}$

$= \frac{1}{49}$

4. a) $(-2)^{-5} = \frac{1}{(-2)^5}$

$= -\frac{1}{32}$

b) $-4^{-2} = -\frac{1}{4^2}$

$= -\frac{1}{16}$

c) $-7^0 = -1$

d) $-5^{-1} = -\frac{1}{5}$

e) $(-3)^{-2} = \frac{1}{(-3)^2}$

$= \frac{1}{9}$

f) $(-4)^{-3} = \frac{1}{(-4)^3}$

$= -\frac{1}{64}$

5. a) $\left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2}$

$= \frac{1}{4}$

b) $\left(\frac{1}{2}\right)^{-2} = 2^2$

$= 4$

c) $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$

$= \frac{8}{27}$

d) $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3$

$= \frac{3^3}{2^3}$

$= \frac{27}{8}$

e) $-\left(\frac{3}{4}\right)^{-2} = -\left(\frac{4}{3}\right)^2$

$= -\frac{4^2}{3^2}$

$= -\frac{16}{9}$

f) $\left(-\frac{3}{4}\right)^{-2} = \left(-\frac{4}{3}\right)^2$

$= \frac{(-4)^2}{3^2}$

$= \frac{16}{9}$

6. a) $2^n = \frac{1}{8}$

$2^n = \frac{1}{2^3}$

$2^n = 2^{-3}$

$n = -3$

b) $4^n = 64$

$4^n = 4^3$

$n = 3$

c) $5^n = 1$

$5^n = 5^0$

$n = 0$

d) $n^{-3} = \frac{1}{27}$

$n^{-3} = \frac{1}{3^3}$

$n^{-3} = 3^{-3}$

$n = 3$

e) $-3^n = -\frac{1}{9}$

$3^n = \frac{1}{3^2}$

$3^n = 3^{-2}$

$n = -2$

f) $(-2)^4 = 16$

$n^4 = (-2)^4$ or 2^4

$n = 2$ or -2

7. 5^{-2} is greater than 10^{-2}. When written in rational form, the numerator is equal to 1, and the denominator of both of these expressions is equal to the base squared. 5 is less than 10; so, 5^2 is less than 10^2, and so 5^{-2} is greater than 10^{-2}.

8. $(-1)^{-100}$ is greater than $(-1)^{-101}$. –100 is an even exponent, making $(-1)^{-100} = 1$, and –101 is an odd exponent, making $(-1)^{-101} = -1$.

Chapter Review, pages 183–186

1. a) Not quadratic. A quadratic relation is of the form $y = ax^2 + bx + c$, with $a \neq 0$. This relation has $a = 0$; so, it is not quadratic.

b)

x	y	First Difference	Second Difference
–3	56		
		–21	
–2	35		4
		–17	
–1	18		4
		–13	
0	5		4
		–9	
1	–4		4
		–5	
2	–9		4
		–1	
3	–10		

This relation is quadratic, as the second differences are constant.

c) Quadratic. Expand the equation.

$y = 2x(x - 5)$

$y = 2x^2 - 10x$

This equation is of the form $ax^2 + bx + c$; so, it is quadratic.

d) Quadratic. This is a symmetrical U-shaped graph.

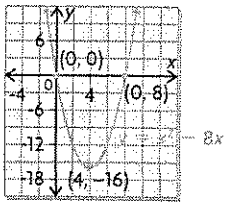
2. Increasing a causes the parabola to become narrower. If $a > 0$, the parabola opens upwards, whereas if $a < 0$, the parabola opens downwards.

If $a > 0$, increasing b causes the parabola to shift left and down, and decreasing b causes it to shift right and down. If $a < 0$, increasing b causes the parabola to shift left and up, and decreasing b causes it to shift right and up.

Increasing c shifts the parabola up.

3. a) First, make a table of values to graph.

x	y
–8	128
–4	48
0	0
4	–16
8	0



i) The parabola is symmetrical about the line $x = 4$.

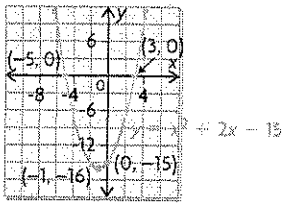
ii) The lowest point on the parabola occurs at (4, –16).

iii) The graph intersects the y-axis at $y = 0$.

iv) The graph intersects the x-axis at $x = 0$ and $x = 8$.

b) First, make a table of values to graph.

x	y
–4	–7
–2	–15
0	–15
2	–16
4	9



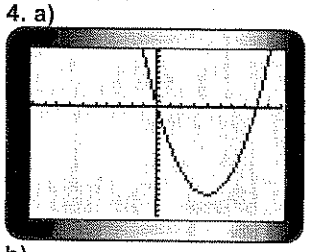
i) The parabola is symmetrical about the line $x = -1$.

ii) The lowest point on the parabola occurs at (–1, –16).

iii) The graph intersects the y-axis at $y = -15$.

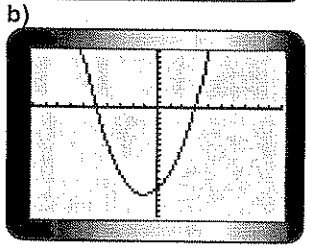
iv) The graph intersects the x-axis at $x = -5$ and $x = 3$.

4. a)



This graph confirms an axis of symmetry at $x = 4$, a vertex at (4, –16), a y-intercept at $y = 0$, and zeros at $x = 0$ and $x = 8$.

b)



This graph confirms an axis of symmetry at $x = -1$, a vertex at (–1, –16), a y-intercept at $y = -15$, and zeros at $x = 3$ and $x = -5$.

5. a) Since the second differences are negative, the parabola must open downwards. Therefore, the parabola has a maximum.

b) The parabola opens downwards, and, because it has zeros, crosses the x-axis. Therefore its vertex must lie above the x-axis, giving its y-coordinate a positive value.

c) By symmetry, the x-coordinate of the vertex occurs exactly halfway between the x-coordinates of each of its zeros. Therefore, the vertex has an x-coordinate of $(-2 + 5) \div 2 = 1.5$.

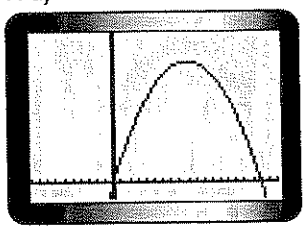
6. For example, create three tables that have (2, 7) as a value. Choose first differences such that the second differences are constant to ensure that each table of values represents a parabola. Determine the other values by adding or subtracting the first differences.

x	y	First Difference	Second Difference
1	2		
		5	
2	7		2
		7	
3	14		2
		9	
4	23		2
		11	
5	34		

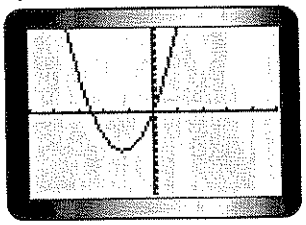
x	y	First Difference	Second Difference
1	1		
		6	
2	7		4
		10	
3	17		4
		14	
4	31		4
		18	
5	49		

x	y	First Difference	Second Difference
1	2		
		5	
2	7		–1
		4	
3	11		–1
		3	
4	14		–1
		2	
5	16		

7. a)



i) The x-intercepts are $x = 0$ and $x = 18$.
ii) The equation of the axis of symmetry is $x = 9$.
iii) The vertex is at (9, 81).
b)



i) The x-intercepts are $x = 0$ and $x = -2.5$.
ii) The equation of the axis of symmetry is $x = -1.25$.
iii) The vertex is at (–1.25, –9.375).

8. For example, the greater the value of a, the narrower the parabola is. Also, if a is positive, then the parabola opens upward, whereas if a is negative, the parabola opens downward.

9. a) Create a table of values to compare profit to the number of snowboards sold.

Snowboards Sold in Thousands (x)	Profit in Hundreds of Thousands of Dollars (y)
0	–60
1	–24
2	0
3	12
4	12
5	0
6	–24
7	–60

The company needs to sell either 2000 or 5000 boards to break even.

b) Since the company breaks even after selling exactly 2000 or 5000 snowboards, then, by the symmetry of quadratic relations, the company must maximize their profits after selling a number of snowboards halfway between these numbers. The company maximizes its profits after selling (2000 + 5000) ÷ 2 = 3500 snowboards.

10. a) Since the x-intercepts are $x = -2$ and $x = 7$, the parabola has the equation $y = a(x - 7)(x + 2)$ in factored form. Substitute the coordinates of the y-intercept into the equation to solve for a.

$$\begin{aligned} y &= a(x - 7)(x + 2) \\ -28 &= a(0 - 7)(0 + 2) \\ -28 &= -14a \\ a &= 2 \end{aligned}$$

The equation is $y = 2(x + 2)(x - 7)$ in factored form.

b) By symmetry, the x-coordinate of the vertex occurs halfway between the x-coordinates of the zeros. Therefore, the vertex has an x-coordinate of $(-2 + 7) \div 2 = 2.5$. Substitute this value into the equation to solve for the y-value of the vertex.

$y = 2(x - 7)(x + 2)$
$y = 2(2.5 - 7)(2.5 + 2)$
$y = 2(-4.5)(4.5)$
$y = -40.5$

The vertex is (2.5, –40.5).

11. a) Since the x-intercepts are $x = 5$ and $x = 9$, the parabola has the equation $y = a(x - 5)(x - 9)$ in factored form. By symmetry, the x-coordinate of the vertex occurs halfway between the x-coordinates of the zeros. So, the vertex has an x-coordinate of $(5 + 9) \div 2 = 7$. Substitute the coordinates of the vertex to solve for a.

$$\begin{aligned} y &= a(x - 5)(x - 9) \\ -2 &= a(7 - 5)(7 - 9) \\ -2 &= -4a \\ a &= 0.5 \end{aligned}$$

The equation is $y = 0.5(x - 5)(x - 9)$ in factored form.

b) Since the x-intercepts are $x = -3$ and $x = 7$, the parabola has the equation $y = a(x + 3)(x - 7)$ in factored form. By symmetry, the x-coordinate of the vertex occurs halfway between the x-coordinates of the zeros. Therefore, the vertex has an x-coordinate of $(-3 + 7) \div 2 = 2$. Substitute the coordinates of the vertex into the equation to solve for a.

$$\begin{aligned} y &= a(x + 3)(x - 7) \\ 4 &= a(2 + 3)(2 - 7) \\ 4 &= -25a \\ a &= -0.16 \end{aligned}$$

The equation is $y = -0.16(x + 3)(x - 7)$ in factored form.

c) Since the x-intercepts are $x = -6$ and $x = 2$, the parabola has the equation $y = a(x - 2)(x + 6)$ in factored form. Substitute the coordinates of the y-intercept into the equation to solve for a.

$$\begin{aligned} y &= a(x - 2)(x + 6) \\ -9 &= a(0 - 2)(0 + 6) \\ -9 &= -12a \\ a &= 0.75 \end{aligned}$$

The equation is $y = 0.75(x - 2)(x + 6)$ in factored form.

d) Since the vertex is a zero, the parabola has the equation $y = a(x - 4)^2$ in factored form. Substitute the coordinates of the y-intercept into the equation to solve for a.

$$\begin{aligned} y &= a(x-4)^2 \\ 8 &= a(0-4)^2 \\ 8 &= 16a \\ a &= 0.5 \end{aligned}$$

The equation is $y = 0.5(x - 4)^2$ in factored form.

e) Since the x-intercepts are $x = -3$ and $x = 3$, the parabola has the equation $y = a(x - 3)(x + 3)$ in factored form. Substitute the coordinates to solve for a.

$$\begin{aligned} y &= a(x-3)(x+3) \\ 20 &= a(2-3)(2+3) \\ 20 &= -5a \\ a &= -4 \end{aligned}$$

The equation is $y = -4(x - 3)(x + 3)$ in factored form.

12. Let x represent the number of \$0.10 increases and y represent the revenue generated. Revenue is equal to the price per ticket, $1 + 0.10x$, multiplied by the number of riders, $12\,000 - 400x$. So, $y = (1 + 0.10x)(12\,000 - 400x)$. The revenue is zero whenever either of the factors equals zero. Set one factor equal to zero.

$$\begin{aligned} 1 + 0.10x &= 0 \\ 0.10x &= -1 \\ x &= -10 \end{aligned}$$

If $x = -10$, there would be 10 decreases in ticket price, and tickets would be free. Thus, no revenue would be generated. Set the other factor equal to zero.

$$\begin{aligned} 12\,000 - 400x &= 0 \\ -400x &= -12\,000 \\ x &= 30 \end{aligned}$$

If $x = 30$, the 30 increases in ticket price would leave the bus with no riders. Thus, in this situation as well, no revenue would be generated. So, the zeros of this relation occur at $x = -10$ and $x = 30$. This is a quadratic relation. By the symmetry of quadratic relations, the maximum revenue will be generated exactly halfway between the zeros. Revenue would be maximized when $x = 10$. Therefore a ticket price of $1 + 0.10(10) = \$2.00$ will maximize revenue.

13. a) The row of light tiles represents $x + 3$, and the column represents $2x - 3$. The dark tiles represent the product of these two binomials. Collecting like terms for the darker tiles shows they represent $2x^2 + 3x - 9$. Therefore, $(x + 3)(2x - 3) = 2x^2 + 3x - 9$.

b) The monomials within the table represent the products of the monomials of their corresponding row and column. The sum of the terms within the table represents the product of the binomials above and beside the table. Thus, $(5x - 6)(3x - 4) = 15x^2 - 38x + 24$.

14. a) $(x + 5)(x + 4) = x^2 + 5x + 4x + 20$
$= x^2 + 9x + 20$

b) $(x - 2)(x - 5) = x^2 - 2x - 5x + 10$
$= x^2 - 7x + 10$

c) $(2x - 3)(2x + 3) = 4x^2 - 6x + 6x - 9$
$= 4x^2 - 9$

d) $(4x + 5)(3x - 2) = 12x^2 + 15x - 8x - 10$
$= 12x^2 + 7x - 10$

e) $(4x - 2y)(5x + 3y) = 20x^2 - 10xy + 12xy - 6y^2$
$= 20x^2 + 2xy - 6y^2$

f) $(6x - 2)(5x + 7) = 30x^2 - 10x + 42x - 14$
$= 30x^2 + 32x - 14$

15. a) $(2x + 6)^2 = (2x + 6)(2x + 6)$
$= 4x^2 + 12x + 12x + 36$
$= 4x^2 + 24x + 36$

b) $-2(-2x + 5)(3x + 4) = -2(-6x^2 + 15x - 8x + 20)$
$= -2(-6x^2 + 7x + 20)$
$= 12x^2 - 14x - 40$

c) $2x(4x - y)(4x + y) = 2x(16x^2 - 4xy + 4xy - y^2)$
$= 2x(16x^2 - y^2)$
$= 32x^3 - 2xy^2$

16. This parabola has zeros at $x = -4$ and $x = 8$; therefore, it has an equation of $y = a(x + 4)(x - 8)$ in factored form. It also has a vertex at (2, 9). Substitute the coordinates of the vertex into the equation to solve for a.

$$\begin{aligned} y &= a(x+4)(x-8) \\ 9 &= a(2+4)(2-8) \\ 9 &= a(6)(-6) \\ 9 &= -36a \\ a &= -0.25 \end{aligned}$$

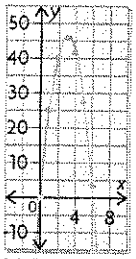
The equation is $y = -0.25(x + 4)(x - 8)$ in factored form. Expand to determine its equation in standard form.

$$\begin{aligned} y &= -0.25(x+4)(x-8) \\ y &= -0.25(x^2 + 4x - 8x - 32) \\ y &= -0.25(x^2 - 4x - 32) \\ y &= -0.25x^2 + x + 8 \end{aligned}$$

The equation is $y = -0.25x^2 + x + 8$ in standard form.

17. Answers may vary, e.g.,

a)



b) The curve of good fit is a parabola, because it is a symmetrical, U-shaped curve.

c) This curve of good fit has x-intercepts at $x = 0$ and $x = 6.1$. So, its equation is $y = ax(x - 6.1)$ in factored form. It also appears to have a vertex at (3.05, 46.5). Substitute this coordinate into the equation to determine a value for a.

$$\begin{aligned} y &= ax(x - 6.1) \\ 46.5 &= a(3.05)(3.05 - 6.1) \\ 46.5 &= -9.3025a \\ a &= -5 \end{aligned}$$

The equation is $y = -5x(x - 6.1)$ in factored form. Expand to convert to standard form.

$$\begin{aligned} y &= -5x(x - 6.1) \\ y &= -5x^2 + 30.5x \end{aligned}$$

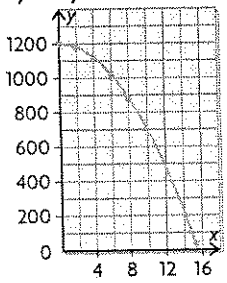
The equation for the curve of best fit is $y = -5x^2 + 30.5x$ in standard form.

d) From the graph of the curve of good fit, the rocket is at a height of about 36 m after 4.5 s.

e) From the graph of the curve of good fit, the rocket is at a height of 20 m after about 0.7 s and after about 5.4 s.

18. Answers may vary, e.g.,

a) & b)



c) The curve of good fit is a parabola, because it appears to be half of a symmetrical, U-shaped curve that can be described by a polynomial of degree 2.

d) Use the standard form, $ax^2 + bx + c$, for the equation. The curve of good fit appears to have a vertex at (0, 1200). Since the vertex occurs on the y-axis, $b = 0$, and its equation is of the form $y = ax^2 + c$. c gives the value of the y-intercept, so $c = 1200$. The point (10, 700) also appears to lie on the curve. Substitute the coordinates of this point into the equation to solve for a.

$$\begin{aligned} y &= ax^2 + 1200 \\ 700 &= a(10)^2 + 1200 \\ 700 &= 100a + 1200 \\ 100a &= -500 \\ a &= -5 \end{aligned}$$

This curve of good fit has the equation $y = -5x^2 + 1200$.

e) From the graph, it appears that the sandbag hits the water after about 15.5 s.

19. a) $2^{-3} = \dfrac{1}{2^3}$

$= \dfrac{1}{8}$

b) $-5^{-1} = -\dfrac{1}{5}$

c) $\left(\dfrac{2}{5}\right)^{-2} = \left(\dfrac{5}{2}\right)^2$

$= \dfrac{5^2}{2^2}$

$= \dfrac{25}{4}$

d) $(-9)^0 = 1$

e) $4^{-3} = \dfrac{1}{4^3}$

$= \dfrac{1}{64}$

f) $-\left(\dfrac{1}{6}\right)^{-2} = -6^2$

$= -36$

20. For example, 3^{-2} is greater because it equals $\left(\dfrac{1}{3}\right)^2$, whose denominator is less than the denominator of $\left(\dfrac{1}{4}\right)^2$.

21. For example, using graphing technology, observe that x^2 (thin line) is greater than 2^x (thick line) when x is between 2 and 4.

Chapter Self-Test, page 187

1. The parabola intersects the x-axis at (–6, 0) and (2, 0); so, these are the zeros. The parabola opens upwards, and the minimum point is at (–2, –4); so, this is the vertex. The axis of symmetry is a line extended vertically from the vertex; thus, it has equation $x = -2$.

2. a) Points (–9, 0) and (19, 0) are zeros on the parabola. The axis of symmetry is a vertical line that passes halfway between the zeros: $(-9 + 19) \div 2 = 5$. Therefore, the axis of symmetry has equation $x = 5$.

b) Since (–9, 0) and (19, 0) are zeros, the equation of the parabola in factored form looks like $y = a(x + 9)(x - 19)$. The vertex occurs at (5, –28), since it must lie on the axis of symmetry. Substitute this point into the equation to solve for a.

$$\begin{aligned} -28 &= a(5 + 9)(5 - 19) \\ -28 &= -196a \\ a &= \frac{1}{7} \end{aligned}$$

The equation for this parabola is $y = \dfrac{1}{7}(x + 9)(x - 19)$ in factored form.

c) $y = \frac{1}{7}(x+9)(x-19)$

$y = \frac{1}{7}(x^2+9x-19x-171)$

$y = \frac{1}{7}x^2 - \frac{10}{7}x - \frac{171}{7}$

The equation is $y = \frac{1}{7}x^2 - \frac{10}{7}x - \frac{171}{7}$ in standard form.

3. a)

x	y	First Difference	Second Difference
–1	1		
		1	
0	2		–6
		–5	
1	–3		–6
		–11	
2	–14		–6
		–17	
3	–31		

This data set can be modelled by a quadratic relation, since the second differences are constant.

b)

x	y	First Difference	Second Difference
0	–4		
		1	
1	–3		2
		3	
2	0		2
		5	
3	5		2
		7	
4	12		

This data set can be modelled by a quadratic relation, since the second differences are constant.

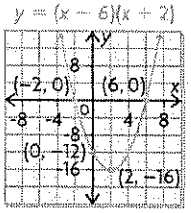
4. a) The graph of this equation has x-intercepts at $x = 6$ and $x = –2$, since $r = 6$ and $s = –2$. Substitute $x = 0$ into the equation to solve for the y-intercept.

$y = (x – 6)(x + 2)$
$y = (0 – 6)(0 + 2)$
$y = (–6)(2)$
$y = –12$

Therefore, the graph has a y-intercept at $y = –12$. By the symmetry of parabolas, the vertex must have an x-coordinate exactly halfway between the x-coordinates of the zeros. Thus, it has an x-coordinate of $(–2 + 6) \div 2 = 2$. Substitute this value into the equation to solve for the y-coordinate of the vertex.

$y = (x – 6)(x + 2)$
$y = (2 – 6)(2 + 2)$
$y = (–4)(4)$
$y = –16$

This graph has a vertex at (2, –16). Sketch the graph.



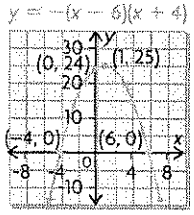
b) The graph of this equation has x-intercepts at $x = 6$ and $x = –4$, since $r = 6$ and $s = –4$. Substitute $x = 0$ into the equation to solve for the y-intercept.

$y = –(x – 6)(x + 4)$
$y = –(0 – 6)(0 + 4)$
$y = –(–6)(4)$
$y = 24$

Therefore, the graph has a y-intercept at $y = 24$.
By the symmetry of parabolas, the vertex must have an x-coordinate exactly halfway between the x-coordinates of the zeros. Thus, it has an x-coordinate of $(–4 + 6) \div 2 = 1$. Substitute this value into the equation to solve for the y-coordinate of the vertex.

$y = –(x – 6)(x + 4)$
$y = –(1 – 6)(1 + 4)$
$y = –(–5)(5)$
$y = 25$

This graph has a vertex at (1, 25). Sketch the graph.



5. a) If $t = 0$ represents 2008, then $t = 10$ represents 2018. Substitute this value into the equation to find the population.

$P = 14t^2 + 820t + 42\,000$
$= 14(10)^2 + 820(10) + 42\,000$
$= 1400 + 8200 + 42\,000$
$= 51\,600$

The population of the city will be 51 600 in 2018.

b) For example, use a table that considers the population in increments of ten years to get a general idea of when the population was about 30 000.

Time in Years (t)	Population [$P = 14(t)^2 + 820(t) + 42\,000$]
–40	$14(-40)^2 + 820(-40) + 42\,000$ = 31 600
–30	$14(-30)^2 + 820(-30) + 42\,000$ = 30 000
–20	$14(-20)^2 + 820(-20) + 42\,000$ = 31 200
–10	$14(-10)^2 + 820(-10) + 42\,000$ = 35 200
0	$14(0)^2 + 820(0) + 42\,000$ = 42 000
10	$14(10)^2 + 820(10) + 42\,000$ = 51 600
20	$14(20)^2 + 820(20) + 42\,000$ = 64 000
30	$14(30)^2 + 820(30) + 42\,000$ = 79 200
40	$14(40)^2 + 820(40) + 42\,000$ = 97 200

The table reveals that at $t = -30$, $P = 30\,000$. Make another table for times close to this year to see in what years the population was close to this value. Define close as differing by less than 350.

Time in Years (t)	Population [$P = 14(t)^2 + 820(t) + 42\,000$]
–35	$14(-35)^2 + 820(-35) + 42\,000 = 30\,450$
–34	$14(-34)^2 + 820(-34) + 42\,000 = 30\,304$
–33	$14(-33)^2 + 820(-33) + 42\,000 = 30\,186$
–32	$14(-32)^2 + 820(-32) + 42\,000 = 30\,096$
–31	$14(-31)^2 + 820(-31) + 42\,000 = 30\,034$
–30	$14(-30)^2 + 820(-30) + 42\,000 = 30\,000$
–29	$14(-29)^2 + 820(-29) + 42\,000 = 29\,994$
–28	$14(-28)^2 + 820(-28) + 42\,000 = 30\,016$
–27	$14(-27)^2 + 820(-27) + 42\,000 = 30\,066$
–26	$14(-26)^2 + 820(-26) + 42\,000 = 30\,144$
–25	$14(-25)^2 + 820(-25) + 42\,000 = 30\,250$
–24	$14(-24)^2 + 820(-24) + 42\,000 = 30\,384$

The population was close to 30 000 for $t = -34$ to $t = -25$. These t-values correspond to the years 1974 to 1983.

6. a) $(2x - 3)(5x + 2) = 10x^2 - 15x + 4x - 6$
$= 10x^2 - 11x - 6$

b) $(3x - 4y)(5x + 2y) = 15x^2 - 20xy + 6xy - 8y^2$
$= 15x^2 - 14xy - 8y^2$

c) $-5(x - 4)^2 = -5(x - 4)(x - 4)$
$= -5(x^2 - 4x - 4x + 16)$
$= -5(x^2 - 8x + 16)$
$= -5x^2 + 40x - 80$

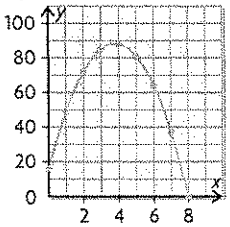
7. Answers may vary, e.g.,
a) The rocket is launched from the top of the tower when $x = 0$. Therefore, the corresponding height of the rocket represents the height of the tower. The tower is 16 m tall.
b) The rocket crashes into the ground when its height above the ground is equal to zero, or $y = 0$. This happens at $x = 8$; so, the rocket is in flight for 8 s.

c)

Time, x (s)	Height, y (m)	First Difference	Second Difference
0	16		
		33	
1	49		–10
		23	
2	72		–10
		13	
3	85		–10
		3	
4	88		–10
		–7	
5	81		–10
		–17	
6	64		–10
		–27	
7	37		–10
		–37	
8	0		

This data set can be represented by a quadratic relation, since the second differences are constant.

d)



e) The curve of good fit appears to have a vertex at about (3.8, 88.2), and one x-intercept at (8, 0). By the symmetry of parabolas, the difference between the x-coordinates of the zeros and the x-coordinate of the vertex should be the same. The difference between the first zero and the vertex is $8 - 3.8 = 4.2$. Therefore, the second zero would occur at $(3.8 - 4.2, 0) = (-0.4, 0)$ if the graph were to be extended. Thus, this curve of good fit has equation $y = a(x + 0.4)(x - 8)$ in factored form. Substitute the value of the vertex into the equation to solve for a.

$y = a(x + 0.4)(x - 8)$
$88.2 = a(3.8 + 0.4)(3.8 - 8)$
$88.2 = a(4.2)(-4.2)$
$88.2 = -17.64a$
$a = -5$

The curve of good fit has equation $y = -5(x + 0.4)(x - 8)$ in factored form. Expand this equation to put it into standard form.

$y = -5(x + 0.4)(x - 8)$
$y = -5(x^2 + 0.4x - 8x - 3.2)$
$y = -5(x^2 - 7.6x - 3.2)$
$y = -5x^2 + 38x + 16$

The equation is $y = -5x^2 + 38x + 16$ in standard form.

f) The maximum height of the rocket occurs at the vertex, which occurs at about 88 m.

8. For example, in either type of modelling, the goal is to find an equation that most accurately fits the data. Modelling with a quadratic relation is generally more difficult, however, because parabolas can take many different forms. For instance, they can be made narrower or wider, translated, or reflected, whereas a line can only be rotated and translated. This also gives quadratic modelling more flexibility, however, allowing it to be used to model a wider variety of relationships.

9. a) $7^{-2} = \frac{1}{7^2}$

$= \frac{1}{49}$

b) $-3^0 = -1$

c) $-\left(\frac{2}{3}\right)^{-4} = -\left(\frac{3}{2}\right)^4$

$= -\frac{3^4}{2^4}$

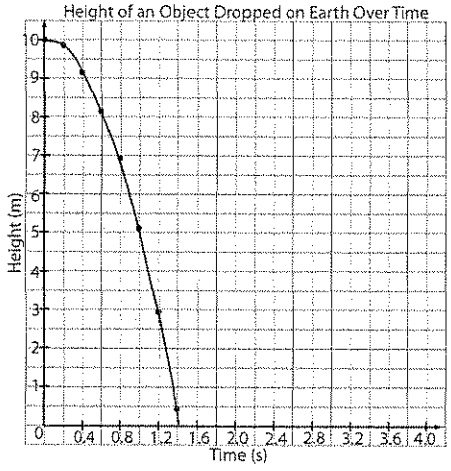
$= -\frac{81}{16}$

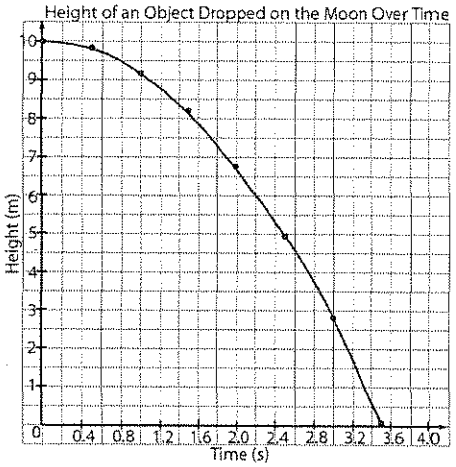
d) $-5^{-3} = -\frac{1}{5^3}$

$= -\frac{1}{125}$

Chapter Task, page 188

A.





B. The data shows that the relation between time and height is quadratic on both Earth and the Moon. The curves of good fit for both graphs resemble half of a U-shape, which identifies them as halves of parabolas. Therefore, the relationship is quadratic.

C. For each graph, the vertex appears to occur exactly when the object is dropped at $t = 0$ and $h = 10$, or (0, 10). This estimate makes sense, because the point at which the object is dropped from should be its highest point. The axis of symmetry is a vertical line that intersects the vertex; thus, it has an equation of $t = 0$. Since the axis of symmetry occurs at $t = 0$, the zeros will be opposite of each other on the time axis. For the object dropped on Earth, the graph reveals there to be a zero at about $t = 1.43$ s, which means there would be another zero at $t = -1.43$ s if the graph were continued. Similarly, for the object dropped on the Moon, the graph shows a zero at $t = 3.51$ s, and so another zero exists at $t = -3.51$ s.

D. Earth: The graph has zeros at $t = -1.43$ s and at $t = 1.43$ s; so, the equation of the curve of good fit is $h = a(t - 1.43)(t + 1.43)$ in factored form. Substitute the coordinates of the vertex into the equation to solve for a.

$h = a(t - 1.43)(t + 1.43)$

$10 = a(0 - 1.43)(0 + 1.43)$

$10 = a(-1.43)(1.43)$

$10 = -2.0449a$

$a = -4.8902$

Therefore, the equation of the curve of good fit for the Earth is $h = -4.8902(t - 1.43)(t + 1.43)$ in factored form. Expand to convert the equation to standard form.

$h = -4.8902(t - 1.43)(t + 1.43)$

$h = -4.8902(t^2 - 1.43t + 1.43t - 2.0449)$

$h = -4.8902(t^2 - 2.0449)$

$h \doteq -4.89t^2 + 10.00$

Therefore, the curve of good fit has equation $h = -4.89t^2 + 10.00$ in standard form.

Moon: The graph has zeros at $t = -3.51$ s and at $t = 3.51$ s; so, the equation of the curve of good fit is $h = a(t - 3.51)(t + 3.51)$ in factored form.

Substitute the coordinates of the vertex into the equation to solve for a.

$$\begin{aligned} h &= a(t-3.51)(t+3.51) \\ 10 &= a(0-3.51)(0+3.51) \\ 10 &= a(-3.51)(3.51) \\ 10 &= -12.3201a \\ a &= 0.8117 \end{aligned}$$

Therefore, the equation of the curve of good fit for the Moon is $h = -0.8117(t-3.51)(t+3.51)$ in factored form. Expand to convert the equation to standard form.

$$\begin{aligned} h &= -0.8117(t-3.51)(t+3.51) \\ h &= -0.8117(t^2 - 3.51t + 3.51t - 12.3201) \\ h &= -0.8117(t^2 - 12.3201) \\ h &\doteq -0.81t^2 + 10.00 \end{aligned}$$

Therefore, the curve of good fit has equation $h = -0.81t^2 + 10.00$ in standard form.

E. The equations for the objects dropped on Earth and the Moon can be rearranged as $h_E = 10.00 - 4.89t^2$ and $h_M = 10.00 - 0.81t^2$ respectively. These equations fit the model $y = H - \left(\frac{g}{2}\right)x^2$ nicely, if y is substituted for h_E or h_M respectively, H is the constant, $\frac{g}{2}$ is the coefficient on t^2, and x is substituted for t. Notice H corresponds to the value 10.00 in both equations, which is indeed the initial height. Thus, for the Earth equation, 4.89 must correspond to $\frac{g}{2}$, making $g_E = 9.78$. Similarly, for the Moon equation, $g_M = 1.62$.

F. Using the values determined in part E,

$$\frac{g_E}{g_M} = \frac{9.78}{1.62}$$

$$\doteq 6.04$$

$$\doteq 6$$

Therefore, gravity on Earth is about 6 times stronger than gravity on the Moon.

Chapter 1–3: Cumulative Review, page 189

1. The correct choice is C.

A. $5x + y = 5(-5) + (-2)$
$= -25 - 2$
$= -27$
$\neq -1$

B. $5x + y = 5(1) + (7)$
$= 5 + 7$
$= 12$
$\neq -1$

C. $3x - 2y = 3(-1) - 2(4)$
$= -3 - 8$
$= -11$

$5x + y = 5(-1) + (4)$
$= -5 + 4$
$= -1$

D. $3x - 2y = 3(1) - 2(-6)$
$= 3 + 12$
$= 15$
$\neq -11$

2. The correct choice is B.

A. $2x + y = 2(5) + 2$
$= 10 + 2$
$= 12$
$\neq 8$

B. $x - 2y = 5 - 2(2)$
$= 5 - 4$
$= 1$

$3x - 4y = 3(5) - 4(2)$
$= 15 - 8$
$= 7$

C. $4y = 4(2)$
$= 8$
$\neq 5 + 5$
$= x + 5$

D. $4x - y = 4(5) - 2$
$= 20 - 2$
$= 18$
$\neq 11$

3. The correct choice is A.

Isolate x:

$5x - 4y + 6 = 0$
$5x = 4y - 6$
$x = 0.8y - 1.2$

Isolate y:

$5x - 4y + 6 = 0$
$5x + 6 = 4y$
$y = 1.25x + 1.5$

Only equation A matches one of these two.

4. The correct choice is D.

Let x represent the monthly fee and y represent the registration fee. From the information given, two equations can be derived: $330 = 3x + y$ and $655 = 8x + y$. By subtracting the two equations I get:

$325 = 5x$
$x = 65$

Now by solving $330 = 3x + y$ when $x = 65$, then,

$y = 330 - 3x$
$y = 330 - 3(65)$
$y = 330 - 195$
$y = 135$

5. The correct choice is C. If the systems are equivalent, they have the same solution. Thus, try evaluating each equation at $(2, -1)$.

A. $2x + y = 2(2) - 1$
$= 4 - 1$
$= 3$
$\neq 9$

B. $6x + 2y = 6(2) + 2(-1)$
$= 12 - 2$
$= 10$
$\neq 12$

C. $x - y = 2 + 1$
$= 3$

$6x = 6(2)$
$= 12$

D. $8x + y = 8(2) - 1$
$= 16 - 1$
$= 15$
$\neq 12$

Only system C has solution $(2, -1)$.

6. The correct choice is B. Let x represent the number of non-students and let y represent the number of students. The number of tickets sold can be represented by $x + y = 771$. The amount of money collected can be represented by $6y + 10x = 6302$. Solve the system for x.
Since $y = 771 - x$ then,

$$\begin{aligned} 6(771 - x) + 10x &= 6302 \\ 4626 - 6x + 10x &= 6302 \\ 4x &= 1676 \\ x &= 419 \end{aligned}$$

There were 419 non-students attending.
7. The correct choice is A. Let x represent the distance in kilometres Brad cycled on trails and y the distance off trails. The total distance can be represented by $x + y = 48$. The time can be represented as $\frac{x}{20} + \frac{y}{12} = 3$.
Solve this system. Since $x = 48 - y$ then,

$$\begin{aligned} \frac{x}{20} + \frac{y}{12} &= 3 \\ \frac{48 - y}{20} + \frac{y}{12} &= 3 \\ 144 - 3y + 5y &= 180 \\ 2y &= 36 \\ y &= 18 \end{aligned}$$

Brad biked 18 km off-trail.
8. The correct choice is C.

A. $M_{AB} = \left(\frac{2+4}{2}, \frac{1-3}{2}\right)$
$= (3,-1)$

B. $M_{CD} = \left(\frac{4-1}{2}, \frac{2-3}{2}\right)$
$= (1.5,-0.5)$

C. $M_{EF} = \left(\frac{-1+7}{2}, \frac{3-7}{2}\right)$
$= (3,-2)$

D. $M_{GH} = \left(\frac{2+8}{2}, \frac{-2-6}{2}\right)$
$= (5,-4)$

The only line segment with midpoint (3, –2) is EF.
9. The correct choice is D. To determine the median from vertex A, calculate the midpoint of BC.

$$M_{BC} = \left(\frac{-2+8}{2}, \frac{1-3}{2}\right)$$
$$= (3,-1)$$

Determine the equation of the line passing through (3, –1) and (2, 7). First, calculate the slope.

$$m = \frac{-1-7}{3-2}$$
$$= -8$$

Only equation D has a slope of –8, so it is the answer.

10. The correct choice is B.

A. $d_{PQ} = \sqrt{(-1-2)^2 + (4-5)^2}$
$= \sqrt{(-3)^2 + (-1)^2}$
$= \sqrt{9+1}$
$= \sqrt{10}$
$\doteq 3.2$

B. $d_{PR} = \sqrt{(-1-1)^2 + (4-2)^2}$
$= \sqrt{(-2)^2 + (2)^2}$
$= \sqrt{4+4}$
$= \sqrt{8}$
$\doteq 2.8$

C. $d_{PS} = \sqrt{(-1+3)^2 + (4-1)^2}$
$= \sqrt{(2)^2 + (3)^2}$
$= \sqrt{4+9}$
$= \sqrt{13}$
$\doteq 3.6$

D. $d_{PT} = \sqrt{(-1+4)^2 + (4-6)^2}$
$= \sqrt{(3)^2 + (-2)^2}$
$= \sqrt{9+4}$
$= \sqrt{13}$
$\doteq 3.6$

Point R is the shortest distance form P.
11. The correct choice is B. The shortest distance from a point to a line is perpendicular, so find the slope that is perpendicular to the given line. Then determine the particular line that each point lies on. Next, determine the intersection of the given line and each calculated one, and calculate the distance between this point and the given point.
First, isolate y:

$$3x + 2y = 6$$
$$y = -\frac{3}{2}x + 6$$

Determine the slope of the perpendicular line and its equation:

$$m = -\frac{3}{2}, \text{ so } m_{perp} = \frac{2}{3}$$
$$y = \frac{2}{3}x + b$$

A. Find the line point $A(0, 0)$ lies on:

$$y = \frac{2}{3}x + b$$

$$0 = \frac{2}{3}(0) + b$$

$$b = 0$$

$$y = \frac{2}{3}x$$

Intersection

$$\frac{2}{3}x = -\frac{3}{2}x + 6 \qquad y = \frac{2}{3}x$$

$$\frac{13}{6}x = 6 \qquad = \frac{2}{3}\left(\frac{36}{13}\right)$$

$$x = \frac{36}{13} \qquad = \frac{24}{13}$$

The point of intersection is $\left(\frac{36}{13}, \frac{24}{13}\right)$.

Distance from the intersection to point A:

$$d_{AP} = \sqrt{\left(\frac{36}{13} - 0\right)^2 + \left(\frac{24}{13} - 0\right)^2}$$

$$= \sqrt{\left(\frac{36}{13}\right)^2 + \left(\frac{24}{13}\right)^2}$$

$$\doteq 3.3$$

B. Find the line point $B(1, 3)$ lies on:

$$y = \frac{2}{3}x + b$$

$$3 = \frac{2}{3}(1) + b$$

$$b = \frac{7}{3}$$

$$y = \frac{2}{3}x + \frac{7}{3}$$

Intersection:

$$\frac{2}{3}x + \frac{7}{3} = -\frac{3}{2}x + 3 \qquad y = \frac{2}{3}x + \frac{7}{3}$$

$$\frac{13}{6}x = \frac{2}{3} \qquad = \frac{2}{3}\left(\frac{4}{13}\right) + \frac{7}{3}$$

$$x = \frac{4}{13} \qquad = \frac{33}{13}$$

The point of intersection is $\left(\frac{4}{13}, \frac{33}{13}\right)$.

Distance from the intersection to point B:

$$d_{BP} = \sqrt{\left(\frac{4}{13} - 1\right)^2 + \left(\frac{33}{13} - 3\right)^2}$$

$$= \sqrt{\left(\frac{-9}{13}\right)^2 + \left(\frac{-6}{13}\right)^2}$$

$$\doteq 0.8$$

C. Find the line point $C(1, -1)$ lies on:

$$y = \frac{2}{3}x + b$$

$$-1 = \frac{2}{3}(1) + b$$

$$b = -\frac{5}{3}$$

$$y = \frac{2}{3}x - \frac{5}{3}$$

Intersection

$$\frac{2}{3}x - \frac{5}{3} = -\frac{3}{2}x + 3 \qquad y = \frac{2}{3}x - \frac{5}{3}$$

$$\frac{13}{6}x = \frac{14}{3} \qquad = \frac{2}{3}\left(\frac{28}{13}\right) - \frac{5}{3}$$

$$x = \frac{28}{13} \qquad = -\frac{3}{13}$$

The point of intersection is $\left(\frac{28}{13}, -\frac{3}{13}\right)$.

Distance from the intersection to point B:

$$d_{CP} = \sqrt{\left(\frac{28}{13} - 1\right)^2 + \left(-\frac{3}{13} + 1\right)^2}$$

$$= \sqrt{\left(\frac{15}{13}\right)^2 + \left(\frac{10}{13}\right)^2}$$

$$\doteq 1.4$$

D. Find the line point $D(-2, 4)$ lies on:

$$y = \frac{2}{3}x + b$$

$$4 = \frac{2}{3}(-2) + b$$

$$b = \frac{16}{3}$$

$$y = \frac{2}{3}x + \frac{16}{3}$$

Intersection

$$\frac{2}{3}x + \frac{16}{3} = -\frac{3}{2}x + 3 \qquad y = \frac{2}{3}x + \frac{16}{3}$$

$$\frac{13}{6}x = -\frac{7}{3} \qquad = \frac{2}{3}\left(-\frac{14}{13}\right) + \frac{16}{3}$$

$$x = -\frac{14}{13} \qquad = \frac{60}{13}$$

The point of intersection is $\left(-\frac{14}{13}, \frac{60}{13}\right)$.

Distance from the intersection to point D:

$$d_{DP} = \sqrt{\left(-\frac{14}{13} + 2\right)^2 + \left(\frac{60}{13} - 4\right)^2}$$

$$= \sqrt{\left(\frac{12}{13}\right)^2 + \left(\frac{8}{13}\right)^2}$$

$$\doteq 1.1$$

Point B is the shortest distance from the line.

12. The correct choice is D.

$r^2 = x^2 + y^2$
$= (-2)^2 + 7^2$
$= 4 + 49$
$= 53$

The equation of the circle is $x^2 + y^2 = 53$.

13. The correct choice is A. A right angle triangle will have two sides whose slopes are perpendicular (negative reciprocals of each other).

$$m_{AB} = \frac{3-5}{-3-2} \qquad m_{BC} = \frac{5+5}{2-6}$$
$$= \frac{-2}{-5} \qquad = -\frac{10}{4}$$
$$= \frac{2}{5} \qquad = -\frac{5}{2}$$

Since $\triangle ABC$ has two negative reciprocal sides, it must be a right triangle.

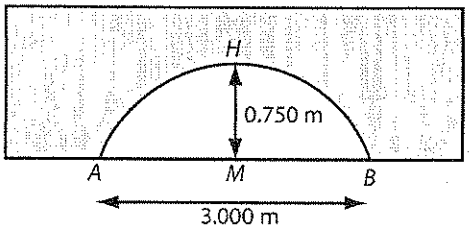
14. The correct choice is D.

$$m_{AB} = \frac{-2-2}{-4+5} \qquad m_{BC} = \frac{2-4}{-5-3}$$
$$= -4 \qquad = \frac{-2}{-8}$$
$$= \frac{1}{4}$$

$$m_{CD} = \frac{1-4}{8-3} \qquad m_{DA} = \frac{1+2}{8+4}$$
$$= -\frac{3}{5} \qquad = \frac{3}{12}$$
$$= \frac{1}{4}$$

Since AB is not parallel to CD, then the shape cannot be a rhombus, a parallelogram or a rectangle. Since it does have one pair of parallel sides it is a trapezoid.

15. The correct choice is B.



Label the diagram as shown. Let X be the diameter. Then:

$AM \times MB = HM \times MX$
$1.5 \times 1.5 = 0.750 \times MX$
$MX = 3$

The diameter is $HM + MX = 0.750 + 3$
$= 3.750$

The radius is equal to half the diameter, therefore the radius is 1.875 m.

16. The correct choice is A. Since the second differences are negative, the parabola opens downward, giving a maximum value. Because there are two zeros, the maximum value is above the x-axis, so it is positive.

17. The correct choice is B. Since the two points have the same y-value, they lie an equal distance away from the axis of symmetry.

Thus, the axis is
$x = (-2 + 4) \div 2$
$x = 2 \div 2$
$x = 1$

When $x = 1$ then,
$y = 3x^2 - 6x - 17$
$y = 3(1) - 6(1) - 17$
$y = 3 - 6 - 17$
$y = -20$

The vertex is $(1, -20)$.

18. The correct choice is B. To determine the equation of the parabola, substitute the zeros into the factored form equation.

$y = (x - r)(x - s)$
$y = (x + 1)(x - 5)$

19. The correct choice is C. To determine the maximum profit, look at the y-value of the vertex. The zeros of the equation are $x = -3$ and $x = 13$.

The axis of symmetry is
$x = (-3 + 13) \div 2$
$x = 5$

When $x = 5$ then,
$P = (-6x + 78)(x + 3)$
$= [-6(5) + 78](5 + 3)$
$= (48)(8)$
$= 384$

The maximum profit is \$384 000.

20. The correct choice is C. Insert the zeros into the factored form equation, and substitute the point given to find the value of a.

$y = a(x - r)(x - s)$
$24 = a(5 + 3)(5 - 7)$
$24 = -16a$
$a = -\frac{3}{2}$

21. The correct choice is D. Insert the zeros and substitute the given point into the factored form to find a.

$y = a(x - r)(x - s)$
$16 = a(32 - 16)(32 - 28)$
$16 = (16)(4)a$
$a = 0.25$

The equation is $y = 0.25(x - 16)(x - 28)$.

22. The correct choice is A.

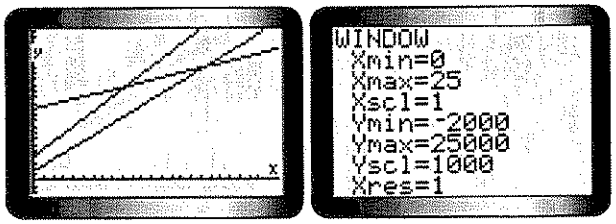
$(3x - 4)(7x + 6) = 3x(7x + 6) - 4(7x + 6)$
$= 21x^2 + 18x - 28x - 24$
$= 21x^2 - 10x - 24$

23. The correct choice is B.

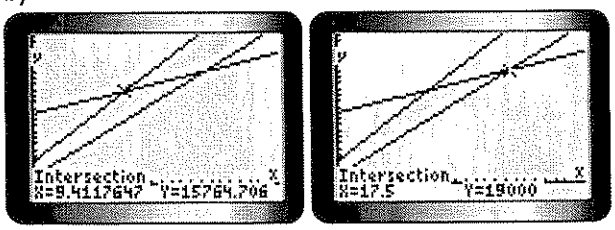
$$\left(\frac{4}{9}\right)^{-2} = \left(\frac{9}{4}\right)^{2}$$
$$= \frac{9^2}{4^2}$$
$$= \frac{81}{16}$$

24. The correct choice is D. Since any number raised to the power of 0 is 1, the answer is D.

25. a) Let t represent the number of years and let C represent the cost. The cost of installation is the fixed cost and the yearly rate is the rate of change.
Gas: $C = 4000 + 1250t$ Electric: $C = 1500 + 1000t$
Geothermal: $C = 12\,000 + 400t$



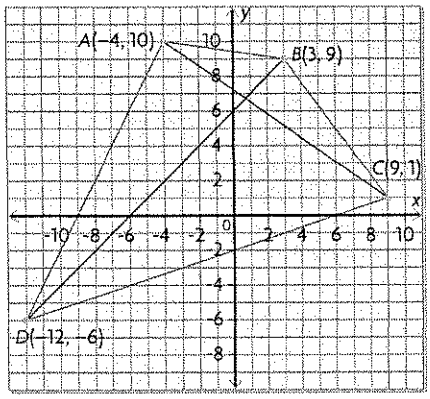
b)



The electric baseboard heaters are the least expensive method of heating for the first 17.5 years. After 17.5 years, the geothermal heat pump is the least expensive. A gas furnace is never the cheapest option, although a gas furnace is more expensive than the baseboard heaters, but it is less expensive that a geothermal heat pump for the first 9.4 years. After 9.4 years, it is the most expensive option.

c) Answers may vary, e.g., the choice depends on how long Jenny and Oliver plan on living in the house. Another factor that they should consider is the uncertainty about gas and electricity prices over time. Geothermal costs will remain relatively constant.

26. a)



Answers may vary, e.g., if all four perpendicular bisectors intersect at the same location, you can draw a circle what passes through all four vertices. The centre of this circle is the point of intersection of the perpendicular bisectors. Determine the equation of the perpendicular bisectors, and then solve the linear system that is formed by two of these equations. Then check to see if the solution satisfies the other two equations.

b) Determine the equation of the perpendicular bisector of AD:

$$m_{AD} = \frac{-6-10}{-12+4}$$
$$= \frac{-16}{-8}$$
$$= 2$$

The perpendicular slope is -0.5.

$$M_{AD} = \left(\frac{-4-12}{2}, \frac{10-6}{2}\right)$$
$$= (-8, 2)$$

$y = mx + b$
$y = -0.5x + b$
$2 = -0.5(-8) + b$
$2 = 4 + b$
$b = -2$

The equation is $y = -0.5x - 2$.

Determine the equation of the perpendicular bisector of DC:

$$m_{DC} = \frac{-6-1}{-12-9}$$
$$= \frac{-7}{-21}$$
$$= \frac{1}{3}$$

The perpendicular slope is -3.

$$M_{DC} = \left(\frac{9-12}{2}, \frac{1-6}{2}\right)$$
$$= (-1.5, -2.5)$$

$y = mx + b$
$y = -3x + b$
$-2.5 = -3(-1.5) + b$
$-2.5 = 4.5 + b$
$b = -7$

The equation is $y = -3x - 7$.

Determine the equation of the perpendicular bisector of CB:

$$m_{CB} = \frac{1-9}{9-3}$$
$$= -\frac{8}{6}$$
$$= -\frac{4}{3}$$

The perpendicular slope is 0.75.

$$M_{CB} = \left(\frac{9+3}{2}, \frac{1+9}{2}\right)$$
$$= (6, 5)$$

$y = mx + b$
$y = 0.75x + b$
$5 = 0.75(6) + b$
$5 = 4.5 + b$
$b = 0.5$

The equation is $y = 0.75x + 0.5$.

Determine the equation of the perpendicular bisector of BA:

$$m_{BA} = \frac{9-10}{3+4}$$
$$= -\frac{1}{7}$$

The perpendicular slope is 7.

$$M_{BA} = \left(\frac{-4+3}{2}, \frac{10+9}{2}\right)$$
$$= (-0.5, 9.5)$$

$y = mx + b$
$y = 7x + b$
$9.5 = 7(-0.5) + b$
$9.5 = -3.5 + b$
$b = 13$

The equation is $y = 7x + 13$.

Calculate the intersection of DC and BA:

$-3x - 7 = 7x + 13$
$-10x = 20$
$x = -2$

Solve for y when $x = -2$.

$y = 7x + 13$
$y = 7(-2) + 13$
$y = -14 + 13$
$y = -1$

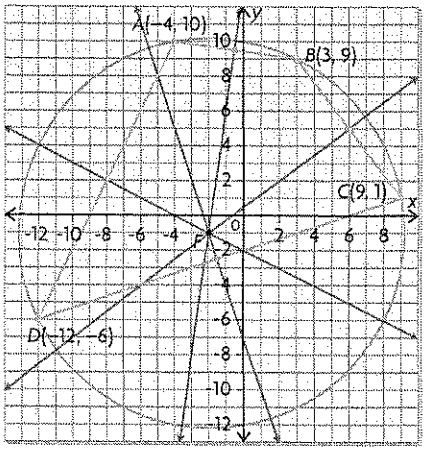
The point of intersection is $(-2, -1)$.

Check this point in the other two equations:

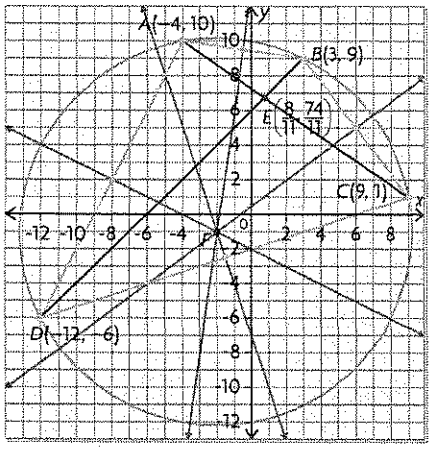
$y = 0.75x + 0.5$	$y = -0.5x - 2$
$y = 0.75(-2) + 0.5$	$y = -0.5(-2) - 2$
$y = -1.5 + 0.5$	$y = 1 - 2$
$y = -1$	$y = -1$

Since all four lines intersect at $(-2, -1)$, then it is possible to draw a circle that passes through all four vertices.

Therefore $ABDC$ is a cyclic quadrilateral.



c)



Determine the equation of the diagonals:

$$m_{AC} = \frac{10-1}{-4-9}$$
$$= -\frac{9}{13}$$

$$y = -\frac{9}{13}x + b$$

$$1 = -\frac{9}{13}(9) + b$$

$$b = \frac{94}{13}$$

The equation of AC is $y = -\frac{9}{13}x + \frac{94}{13}$.

$$m_{DB} = \frac{-6-9}{-12-3}$$
$$= \frac{-15}{-15}$$
$$= 1$$
$y = 1x + b$
$9 = 3 + b$
$b = 6$

The equation of DB is $y = x + 6$.

The intersection of these lines is

$$x + 6 = -\frac{9}{13}x + \frac{94}{13}$$
$$13x + 78 = -9x + 94$$
$$22x = 16$$
$$x = \frac{8}{11}$$

Solve for y:

$$y = x + 6$$
$$= \frac{8}{11} + 6$$
$$= \frac{74}{11}$$

Calculate the lengths of AE, EC, BE and ED.

$$d_{AE} = \sqrt{\left(\frac{8}{11}+4\right)^2 + \left(\frac{74}{11}-10\right)^2}$$
$$= \sqrt{\left(\frac{52}{11}\right)^2 + \left(-\frac{36}{11}\right)^2}$$
$$\doteq 5.749\,595\,746 \text{ units}$$

$$d_{EC} = \sqrt{\left(\frac{8}{11}-9\right)^2 + \left(\frac{74}{11}-1\right)^2}$$
$$= \sqrt{\left(-\frac{91}{11}\right)^2 + \left(\frac{63}{11}\right)^2}$$
$$\doteq 10.061\,792\,56 \text{ units}$$

$$d_{BE} = \sqrt{\left(\frac{8}{11}-3\right)^2 + \left(\frac{74}{11}-9\right)^2}$$
$$= \sqrt{\left(-\frac{25}{11}\right)^2 + \left(-\frac{25}{11}\right)^2}$$
$$\doteq 3.214\,121\,733 \text{ units}$$

$$d_{ED} = \sqrt{\left(\frac{8}{11}+12\right)^2 + \left(\frac{74}{11}+6\right)^2}$$
$$= \sqrt{\left(\frac{140}{11}\right)^2 + \left(\frac{140}{11}\right)^2}$$
$$\doteq 17.999\,081\,7 \text{ units}$$

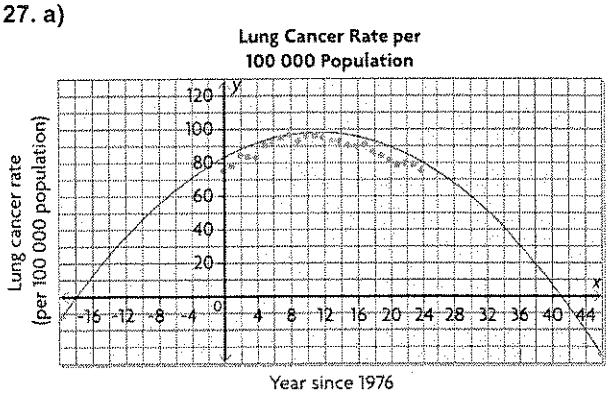
$$AE \times EC = \frac{7000}{121}$$
$$\doteq 57.851\,239\,7 \text{ units}$$

$$BE \times ED = \frac{7000}{121}$$
$$\doteq 57.851\,239\,7 \text{ units}$$

Since $AE \times EC = BE \times ED$, then AC and BD are chords of the same circle.

d) Any square, rectangle or isosceles trapezoid is cyclic. Kites can be cyclic if and only if they have two right angles.

27. a)



b) Answers may vary, e.g., I estimate the zeros of the graph are (–18, 0) and (41, 0). A point that appears on the curve is (17, 95).

$y = a(x-r)(x-s)$
$y = a(x+18)(x-41)$
$95 = a(17+18)(17-41)$
$95 = (35)(-24)a$
$95 = -840a$

$$a = -\frac{95}{840}$$
$$a \doteq -0.113\,095$$

I estimate the equation to be
$y = -0.113\,095(x+18)(x-41)$.

c) A regression of this graph gives the equation $y = -0.1358x^2 + 3.0691x + 77.309$. To compare the two equations, expand the estimation into standard form.

$y = -0.113\,095(x + 18)(x - 41)$
$y = -0.113\,095(x^2 - 41x + 18x - 738)$
$y = -0.113\,095(x^2 - 23x - 738)$
$y = -0.113\,095x^2 + 2.601\,185x + 83.464\,11$

The curves are similar. Thus, the curve of best fit is a good approximation.

d) If the trend continues, lung cancer rates in Canadian males will continue to decrease.